Journal of
Numerical Cognition

PsychOpen GOLD

Empirical Research

Children's Early Understanding of the Successor Function

Theresa Elise Wege[1], Camilla Gilmore[1], Matthew Inglis[1]

[1] *Centre for Mathematical Cognition, Loughborough University, Loughborough, United Kingdom.*

Journal of Numerical Cognition, 2025, Vol. 11, Article e11371, https://doi.org/10.5964/jnc.11371

Received: 2023-02-13 • **Accepted:** 2025-03-13 • **Published (VoR):** 2025-10-22

Handling Editor: Geetha Ramani, University of Maryland, College Park, MD, USA

Corresponding Author: Camilla Gilmore, Centre for Mathematical Cognition, Loughborough University, LE11 3TU, Loughborough, UK. E-mail: c.gilmore@lboro.ac.uk

Supplementary Materials: Data, Materials [see Index of Supplementary Materials]

Abstract

Children learn the cardinalities of the first numbers one, two, three and four before they learn how counting tracks cardinality for all numbers. It may be that when children start to understand counting, they also discover how numbers relate to one another in a structured number system. Do children who understand that the cardinality of a set is the last number assigned after counting each item (cardinal principle knowledge) also understand that each number represents the cardinality of the set created by adding one to an empty set for every count it takes to reach that number (a recursive understanding of the successor function)? We tested this by assessing children's early recursive understanding of the successor function in relation to their cardinality knowledge. Children who were not yet cardinal principle knowers already demonstrated a recursive understanding of the successor function within the limits of their cardinality knowledge. Our findings suggest that children have some structural knowledge of the number system before learning how counting tracks cardinality. We discuss how continued counting practice may eventually allow children to expand this knowledge across all numbers.

Keywords

number, counting, cardinality, mathematics, numerical cognition, conceptual development

Non-Technical Summary

Background

Learning the meaning of the number system is a challenge for young children. They must discover how counting gives information about the number of objects in a set (*cardinal principle*) and how adding one item to a set is associated with moving one place along the count sequence (*successor function*).

Why was this study done?

Learning this takes many months and the processes which underlie it are as yet unknown. We developed a new task that sheds light on this development.

What did the researchers do and find?

Three- and four-year-old children were asked to create sets of different numbers of objects in two conditions. In one version of the task children could see the whole set of objects at the same time and this gave us insight into children's understanding of the cardinal principle. In the other version they could only see one object at a time and this gave us insight into children's

understanding of the successor function. We found that most young children's understanding of the cardinal principle and the successor function were equivalent. In other words, if they could create a set of up to two objects when the objects were visible then they could also create a set of up to two objects when they could only see one at a time.

What do these findings mean?
This suggests that understanding of cardinality is not a pre-requisite of understanding the successor function and that learning the meaning of the number system is a continuous and progressive process.

Highlights

- Children may discover the structure of the number system by jointly understanding the cardinal principle and the successor function.
- We assessed understanding of the successor function in relation to cardinality knowledge with a novel Give-*N* task.
- Children who do not yet know the cardinal principle understand the successor function for all numbers they know the cardinality of.
- Children may discover the structure of the number system progressively and through continuous practice.

As young children learn to count, they need to learn both how numbers relate to one another – the structure of the number system – and also how numbers represent the cardinality of sets. Already at the age of 2- to 3-years-old, children know that the numbers go in an ordered sequence, "one, two, three, four, five, six…". But they do not yet understand how the numbers represent cardinalities of sets and how the number sequence reflects the structure of the number system in which the next number is always one more (Sarnecka & Carey, 2008). At around 3- to 4-years-old and over a span of several months, children learn that the first small numbers represent cardinalities of sets, e.g., "three" refers to a set with the cardinality |{a, b, c}|. Only after learning the cardinalities of a subset of the first numbers, children begin to understand how the number sequence tracks cardinality and start to use counting to reason about sets (Le Corre & Carey, 2007; Wynn, 1992). As children start to use counting in this way, they must understand the structure of the number system to some extent. They may understand the *cardinal principle*: the cardinality of a set is the last number assigned after counting each item in the set. They may also understand the *successor function:* for every number in the ordered sequence the next number represents the cardinality of a set with one more item. It has been theorized that children jointly understand the cardinal principle and the successor function in a moment of conceptual induction about the structure of the number system (Carey, 2000, 2009). However, it may be that rather than a single moment of conceptual induction, understanding of the successor function and the cardinal principle represent a continuous process that scaffolds discovery of the structure of the number system (Barner, 2017). The current study attempts to distinguish these accounts.

Learning the Cardinality of One, Two, Three, Four and the Cardinal Principle

Until children understand the cardinal principle, counting does not yet offer a meaningful way to assign number words to cardinalities of sets (Gelman & Gallistel, 1978; Le Corre & Carey, 2007). However, even before children understand the cardinal principle, they learn how a subset of the first small numbers, usually "one" through "three" or "four" represent cardinalities (Wynn, 1990, 1992). At this point, they are able to understand cardinality in a local sense applied to specific numbers before they understand this in a global sense (Batchelor et al., 2015). They likely do so by forming associative mappings in long-term memory in the form of "three" being associated with a set containing three items, {a, b, c} (Carey & Barner, 2019; Le Corre & Carey, 2007; Sullivan & Barner, 2014). To determine the cardinality of any set of objects, these mappings can be retrieved for comparison via one-to-one correspondence, e.g. |{toy, other toy, another toy}| is equal to |{a, b, c}| and hence is a set of "three" toys (Cheung & Le Corre, 2018). Children's behaviour suggests that they acquire these associative mappings slowly and sequentially; first for the number "one", then "two" etc. (Wynn, 1992). Because children only know the cardinalities of a subset of numbers at this stage they are often referred to as subset-knowers.

Once children become subset-knowers, they can succeed at some versions of the Give-*N* task. When asked “can you give me *n* blocks?”, subset-knowers succeed only up to the largest number *n* that they know the cardinality of. A cardinal three-knower, for instance, succeeds at giving the correct number of blocks when asked for one, two and three but grabs a bunch of blocks for any other number (Sarnecka & Lee, 2009; Wynn, 1990). At this point their behaviour is consistent with local knowledge of the cardinal principle limited to numbers up to three. It has been proposed that to learn the cardinalities of the numbers up to four, children can rely on their ability to form associative mappings based on tracking and recognizing all the items in a set (Feigenson et al., 2004; Hyde, 2011; Piazza et al., 2011). Because this ability is limited to about four items, children only succeed at producing larger cardinalities, such as six, on the Give-*N* task once they start using counting to track cardinality. They must understand the cardinal principle of counting: that each item added to the set needs to be counted and that the final count represents the cardinality of the set. Children who succeed at giving larger numbers (such as six) on the Give-*N* task do so because they can use counting meaningfully, and they can use counting meaningfully because they have global knowledge of cardinality applied to all numbers in their count list, i.e. they are cardinal principle (CP) knowers (Sarnecka, 2021; Sarnecka et al., 2015).

Learning the Structure of the Number System: A Joint Discovery With the Cardinal Principle?

The transition from cardinal subset-knower to CP-knower has been a focus point for theories on how children come to understand the structure of the number system (e.g., Barner, 2017; Carey & Barner, 2019; Davidson et al., 2012; Geary et al., 2018; Le Corre & Carey, 2007; Paliwal & Baroody, 2020; Sarnecka, 2021; Sarnecka & Carey, 2008; Sella et al., 2021; Shusterman et al., 2016; Simon et al., 2023; Spaepen et al., 2018; Wynn, 1990).

In her influential proposal on the origin of concepts, Carey (2000, 2009) theorized that becoming a CP-knower represents a moment of conceptual induction in which children discover the structure of the number system. Specifically, she proposed that children unlock counting as a powerful tool by gaining insight into the successor function between all numbers in the ordered sequence. After children have learned the first small numbers via associative mappings, they notice that adding one item to the set changes the cardinality of the set to the next number in the sequence: “one” is $|\{a\}|$, “two” is $|\{a, b\}|$ but “two” also is one more than one, $|\{a\} \cup \{b\}|$ and “three” is again not just the cardinality $|\{a, b, c\}|$ but also the cardinality of the set created by adding one to one to another one $|\{a\} \cup \{b\} \cup \{c\}|$, and therefore also the cardinality of the set created by adding one to a set with the cardinality two $|\{a, b\} \cup \{c\}|$. By connecting this insight about how cardinalities are related by set operations to the order of the number sequence, children realize that “five” must be the fifth successor of the cardinality of the empty set $|\{a\} \cup \{b\} \cup \{c\} \cup \{d\} \cup \{e\}| = S^5(|\varnothing|)$ and “six” must again be just one more than “five” $6 = S(5)$ and so forth. Here we use S to denote the successor function, so that $S(0) = 1$, $S(1) = 2$, …, $S(n) = n + 1$, etc. Because 0 is the cardinality of the empty set we can also write $|\varnothing| = 0$, $S(|\varnothing|) = 1$, $S(S(|\varnothing|)) = 2$, etc. Mathematicians denote $S(S(|\varnothing|))$ as $S^2(|\varnothing|)$, and in general the nth successor of $|\varnothing|$ is n, so that $S^n(|\varnothing|) = n$. The discovery of the structural analogy between set operations and the ordered number sequence fuels the induction that counting produces sets of a certain cardinality by successively adding one item when moving to the next number in the sequence (see also Gentner, 2010; Le Corre & Carey, 2007; Lipton & Spelke, 2006; Sarnecka & Carey, 2008; Wynn, 1992).

Carey’s (2000, 2009) account predicts that as children become CP-knowers they also understand that for every number in the ordered sequence the next number represents a set with one more item, $S(n) = n + 1$, and hence that the cardinality of any number is the set created by adding one item to an empty set for every count it takes to reach that number in the ordered sequence, $n = S^n(|\varnothing|)$. Mathematically, $S^n(|\varnothing|) = n$ and $S(n) = n + 1$ both follow from the definition of the successor function, which is fundamental to the structure of the number system. Adding one to nothing makes “one”, adding one more makes “two”, then “three” … “three-million”, “three-million-and-one” … and so on indefinitely.

There are two predictions we can derive from the proposal that children begin to understand counting and become CP-knowers because they understand the structure of the number system (Davidson et al., 2012; Le Corre & Carey, 2007). First, we would expect all children who are CP-knowers to have knowledge of the successor function across all numbers in their known number sequence, i.e. to have global understanding of the successor function. Second, we would expect children who are not yet cardinal principle knowers to lack an understanding of the successor function in both the global sense and a local sense (i.e. they lack an understanding of the successor function even for those numbers they know the cardinality of). Evidence of children's behaviour has accumulated to suggest neither of these expectations may be the case.

Understanding the Structure of the Number System Only After Learning the Cardinal Principle?

Only after months or even years of having known the cardinal principle do children appear to fully understand that $S(n) = n + 1$ (Davidson et al., 2012; Schneider et al., 2021a, 2021b; Spaepen et al., 2018). One source of evidence is the inconsistent performance of CP-knowers on the unit-task. In the unit-task, children are presented with an occluded set of objects, told the cardinality of the set is n, shown the addition of one object and then asked if the cardinality of the set is now $n + 1$ or $n + 2$ (Sarnecka & Carey, 2008; Sarnecka & Gelman, 2004; Sella & Lucangeli, 2020). Although CP-knowers can use counting to successfully produce sets of five and six on the Give-*N* task, many CP-knowers struggle to answer "There are five blocks in the box. Watch me put one more block in the box. Are there now six or seven blocks?". It may in fact, take years after children have become CP-knowers before they understand that all numbers n have a successor $n + 1$ (Cheung et al., 2017; Chu et al., 2020).

These findings have been taken to contradict Carey's account, in which becoming a CP-knower represents a moment of conceptual induction and discovery of the structure of the number system. Instead, it has been proposed that becoming a CP-knower may simply equip children with a procedure and practice that allows them to eventually discover the structure of the number system (Barner, 2017; Davidson et al., 2012; Schneider et al., 2021a; Spaepen et al., 2018). Potentially, only once children start using counting to produce and reason about sets are they able to notice how adding one item to a set changes the cardinality of the set not only to a different number but to always exactly the next number in the number sequence (Spaepen et al., 2018). This still leaves the possibility that CP-knowers' failure on the unit-task is not a failure to understand that $S(n) = n + 1$. Instead, it may be a failure to retrieve the order of the number in the sequence when having to count on from a large number rather than counting up from one (Le Corre, 2014; Sella et al., 2020; Spaepen et al., 2018). This is because the unit-task requires that children can count on from the prompted number which, as Schneider et al. (2021b) show, is not always the case, even for CP-knowers.

We can distinguish two aspects of the successor function. First, a subsequent meaning of the successor function, as represented by $S(n) = n + 1$. In addition to this, an important feature of the structure of the number system is that the cardinality of any set is the number created by adding one to the cardinality of the empty set for every count it takes to reach that number in the ordered sequence, i.e. $n = S^n(|\varnothing|)$. We call this the recursive meaning of the successor function. It remains a possibility that learning to count represents discovery of the structure of the number system because it represents the moment children understand that cardinality is produced by successively adding one for each number counted and the last number counted represents the cardinality of the set. The unit task measures children's understanding of the subsequent meaning of the successor function but not the recursive meaning. In our view, testing whether a child understands that $n = S^n(|\varnothing|)$ provides a more general test of number structure understanding than testing whether they understand $S(n) = n + 1$, for two reasons. First, anyone who understands that $n = S^n(|\varnothing|)$ must also implicitly understand that $S(m) = m + 1$, at least for $m < n$; and second, testing whether children understand $n = S^n(|\varnothing|)$ does not require that children can count on from a large number. We therefore ask: does learning the cardinal principle coincide with learning that $n = S^n(|\varnothing|)$?

Understanding the Structure of the Number System Already Before Learning the Cardinal Principle?

Children who are not yet CP-knowers may nevertheless understand the structure of the number system within the subset of numbers that they know the cardinalities of, i.e. they may have local understanding of the successor function even if they do not have global understanding. Evidence regarding this issue is very limited. Schneider and colleagues (2021a) used the Give-*N* task to classify children not only as CP-knowers and subset-knowers but also further classified subset-knowers based on their largest known number into one-knowers, two-knowers, three-knowers and four-knowers. Similar to young CP-knowers, subset-knowers at all knower-levels failed the unit-task. They appeared to be guessing when asked after the addition of one item whether the set now contained “$n + 1$ or $n + 2$” if n was “four” or “five”. Surprisingly however, subset-knowers answered significantly above chance when numbers were within their known number range, e.g., three-knowers were above chance on the unit task when n was “one”, “two” or “three”. While Carey (2000, 2009) proposed the insight that “one” is $|\{a\}|$, “two” is one more than one $|\{a, b\}| = |\{a\} \cup \{b\}|$ and “three” is one more again $|\{a, b, c\}| = |\{a, b\} \cup \{c\}| = |\{a\} \cup \{b\} \cup \{c\}|$ to happen jointly with the discovery of the cardinal principle, Schneider and colleagues’ (2021a) findings suggest that children may already understand aspects of the structure of the number system as they learn the first numbers.

There are two reasons why Schneider and colleagues’ (2021a) results do not allow us to draw strong conclusions about whether subset-knowers understand the structure of the number system for all numbers that they know the cardinality of. Firstly, the unit-task is not capable of probing whether subset-knowers understand the successor function for all their known numbers. When asked about their highest known number, children may only incidentally answer correctly by selecting the only known number out of the two options, e.g., three-knowers may have answered correctly for “is it now three or four?” because to them only three is a number that refers to an exact cardinality. Secondly, while the unit-task probes children’s understanding of $S(n) = n + 1$ (i.e. subsequent understanding of the successor function), to test Carey’s account it is also highly relevant whether subset-knowers also already understand that $n = S^n(|\varnothing|)$ (i.e. recursive understanding of the successor function; see also Barner, 2017). Do children who know that three is a set with the cardinality three also know that know that “three” $= |\{a, b, c\}| = |\{a\} \cup \{b\} \cup \{c\}|$. If children who only know the cardinalities of the first small numbers already understand that those cardinalities are produced by successively adding one to the previous cardinality, it may not require a moment of conceptual induction but simply continued practice for them to learn that this successor function is fundamental to the structure of the number system. We therefore ask: do subset-knowers already understand that $n = S^n(|\varnothing|)$ for all n they know the cardinality of? In other words, if they show a local understanding of cardinality, do they also show a local understanding of the recursive meaning of the successor function?

The Current Study

Is there a moment of conceptual induction after which children understand both the structure of the number system and the cardinal principle (Carey, 2000, 2009) or does understanding of the structure of the number system and the cardinal principle represent a continuous practice (Barner, 2017)? In the current study, we contributed to this debate by addressing two open questions regarding how and when children discover fundamental aspects of the structure of the number system:

> 1) Does learning the cardinal principle coincide with learning the recursive meaning of the successor function, $n = S^n(|\varnothing|)$?

And if it does not:

> 2) Do subset-knowers have a local understanding of the recursive meaning of the successor function and understand that $n = S^n(|\varnothing|)$ for all n they know the cardinality of?

To answer these questions, we assessed children’s knowledge of the cardinality of the first number words and the cardinal principle of counting and then assessed their knowledge of the successor function correspondingly. We used the Give-*N* task to classify children as CP-knowers and as subset-knowers of particular knower-levels. The Give-*N* task assesses children’s knowledge of the cardinality of a set referred to by the requested number word. In line with previous research (Le Corre & Carey, 2007), we infer children’s knowledge of the cardinal principle based on whether their behaviour on the task is consistent with this. The standard Give-*N* task does not, however, assess knowledge of the successor function. Children produce sets by taking items from an openly visible pile and place them into another openly visible spot (e.g. on a plate to feed the stuffed animal with “three” oranges). They do not have to know that to produce a set of with the cardinality “three” they have to successively add one item, then another and another. Instead, they can rely on visual cues about the “threeness” of the set in front of them (Clements et al., 2019; Piazza et al., 2011). Hence, to assess children’s knowledge of the successor function in relation to their knower-level, we also presented children with a modified successor Give-*N* task. In the successor Give-*N* task, children had to produce sets under occlusion by successively adding one item at a time. Analogously to the standard Give-*N* task, we infer children's knowledge of the successor function based on whether their behaviour is consistent with this. Doing this successfully for any number n evidenced children's understanding of the recursive meaning of the successor function, $n = S^n(|\varnothing|)$.

Using the standard cardinality Give-*N* task and the successor Give-*N* task allowed us to also score children's highest known number for both tasks. This allowed us to test the diverging predictions related to our two main research questions:

> 1) Does learning the cardinal principle coincide with learning the recursive meaning of the successor function, $n = S^n(|\varnothing|)$?

If so, CP-knowers who succeed at giving all numbers on the cardinality Give-*N* task also succeed for all numbers on the successor Give-*N* task while subset-knowers fail to give the numbers they succeed at in the cardinality Give-*N* task in the successor Give-*N* task. If learning the cardinal principle *does not* coincide with learning that $n = S^n(|\varnothing|)$, we may either observe that some, many or even all CP-knowers fail on the successor Give-*N* task and do not perform differently to subset-knowers. This would further support the conclusions that understanding of the cardinal principle precedes understanding of multiple crucial notions of the successor function. Alternatively, we may find that learning the cardinal principle *does not* coincide with learning that $n = S^n(|\varnothing|)$ by observing subset-knowers' success on the successor Give-*N* task, leading us to our second question.

> 2) Do subset-knowers have a local understanding of the recursive meaning of the successor function and understand that $n = S^n(|\varnothing|)$ for all n they know the cardinality of?

If so, subset-knowers who succeed at giving some of the first numbers on the cardinality Give-*N* task also succeed at giving some of the first numbers in the successor Give-*N* task. It may be that subset-knowers succeed on the successor Give-*N* task according to their knower-level as determined via the cardinality Give-*N* task. This would indicate that as children first learn the cardinality of a number, they also understand how this cardinality can be produced by successively adding one to an empty set (i.e. recursive understanding of the successor function). Instead of first learning an associative mapping between "three" and $|\{a, b, c\}|$ and then discovering that "three" is $|\{a\} \cup \{b\} \cup \{c\}|$, these insights may be closely linked or even identical. On the other hand, we may observe that subset-knowers succeed on the successor Give-*N* task but that success is limited below their knower-level as determined via the cardinality Give-*N* task. Such a finding would suggest that only after learning the associative mappings representing the cardinality of a number do children discover the successor function between those mappings, e.g. a three-knower may already know that "two" is the cardinality created by adding one and another one $|\{a\} \cup \{b\}|$, but has not discovered this fact for the only recently learned number "three" and knows "three" only as the cardinality of a set with three items $|\{a, b, c\}|$ but not as the set created by adding one and another one and another one $|\{a\} \cup \{b\} \cup \{c\}|$.

Method

Anonymized data are freely available: https://osf.io/6bf9w/. The experiment was not preregistered: the composition of a sample into subset and CP-knowers is extremely difficult to predict and to achieve any preregistered thresholds requires recruiting and screening an uneconomically large number of children. Instead, we decided to recruit as many children as possible within the fixed timeframe of the study. As we laid out above, there were several different possibilities regarding children's performance on the successor Give-*N* task in relation to their performance on the cardinality Give-*N* task. We did not adopt a frequentist hypothesis testing approach in this study but rather aimed to quantify observed (in)consistencies between children's performance on both tasks. We further developed a novel analysis technique to quantify evidence in relation to our research questions while directly taking into account the observed uncertainty around individual children's performance on both tasks.

Participants

We recruited an opportunity sample of 3- and 4-year-old children. Children were recruited from 3 avenues: 1) parents bringing their children to their workplace at Loughborough University, 2) two local nurseries and 3) a children's science fair at the University of Nottingham. We aimed to collect at least 40 usable observations with similar numbers of subset- and CP-knowers. The final sample consisted of 40 children (age in years: *M* = 4.10, *SD* = 0.46, *Min* = 3.01, *Max* = 4.80). In compliance with the European General Data Protection Regulation (GDPR), no additional demographic information was collected.

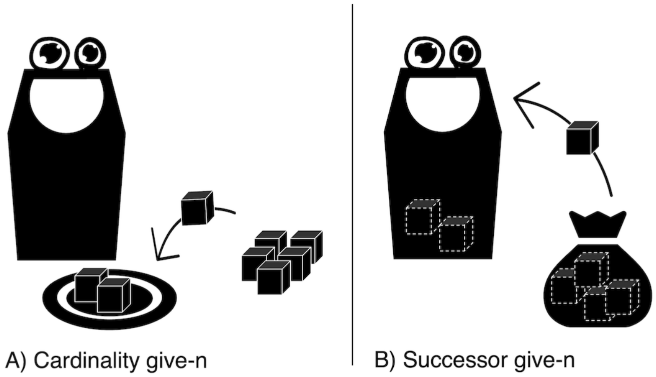
Based on scores derived from the Bayesian model, we achieved similar numbers of CP-knowers (*n* = 21, age in years: *M* = 4.31, *SD* = 0.27, *Min* = 3.83, *Max* = 4.80) and subset-knowers (*n* = 19, age in years: *M* = 3.87, *SD* = 0.51, *Min* = 3.01, *Max* = 4.62). Based on "two out of three" scores, one child (age in years = 4.03) classified as a subset-knower would instead be classified as CP-knower. We explain details about this scoring below. Seven additional children participated but were excluded. They either only completed one of the tasks (*n* = 6) or failed on all trials of the cardinality Give-*N* task indicating that they were not yet subset-knowers (*n* = 1).

Materials

Both Give-*N* tasks were presented with "Benny Bin" and his favourite food, blocks. "Benny Bin" was an opaque table bin with a swing lid (2L volume, 22cm tall, 15cm wide) and googly eyes glued to its side. "Benny Bin's" favourite food were 15 wooden blocks (2cm x 2cm x 2cm). In the cardinality Give-*N* task, we additionally used a plastic plate (15cm diameter) and in the successor Give-*N* task we additionally used an opaque fabric bag (27cm tall, 20cm wide) with a narrow opening that was held by an elastic band.

Figure 1

Set-Up of A) the Cardinality Give-N Task and B) the Successor Give-N Task



Note. In the cardinality Give-*N* all blocks were visible at all times. Children produced sets by placing blocks on a plate in any way they liked. In the successor Give-*N* any sets of blocks were occluded. Children produced sets by taking blocks out from the bag and dropping them into the bin one at a time.

Procedure

Each child participated in both tasks, the cardinality Give-*N* task and the successor Give-*N* task (Figure 1). The order of the tasks was counterbalanced across children. Basic procedures for both tasks were identical. The instruction for both tasks was to feed "Benny Bin" the right number of blocks. Each task started with the experimenter demonstrating how to feed the blocks. The experimenter asked the child to count as she fed 6 blocks to Benny Bin and then asked "how many blocks, did I feed Benny Bin?". After answering the counting questions, children were told that it was now their turn to feed the blocks. In both tasks, children were asked to give the numbers 1, 2, 3, 4 and 6 three times each for a total of 15 trials (most children did not complete all trials, but to be included in the final sample they had to give each number at least twice). The experimenter only instructed children to “Let me know when it’s right” and did not prompt children to count. We recorded the number of blocks given in each trial, whether children spontaneously counted and whether they counted correctly. The two tasks were largely identical in the procedure with the only differences being how the blocks had to be fed to Benny Bin and whether blocks were visible simultaneously.

Cardinality Give-N Task

In this task, children were given a pile of the 15 wooden blocks on the table in front of them. They were then asked to give a certain number of blocks to “Benny Bin” by placing them on the plate in front of “Benny Bin” (Figure 1A). They were not required to place blocks one at a time but could do so in any way they liked. All blocks were visible at all times. After each trial, blocks from the plate were returned to the pile in front of the child.

Successor Give-N Task

In this task, children were given the 15 wooden blocks contained in the opaque fabric bag. They were then asked to give a certain number of blocks to “Benny Bin” by putting blocks into the bin through the swing lid (Figure 1B). Neither the blocks in the bin nor the blocks in the bag were visible. The experimenter ensured that children would take blocks out from the bag and drop into the bin one at a time. After each trial blocks from inside the bin were returned to the bag.

Scoring

The two Give-*N* tasks were scored independently. We scored children’s performance as the highest known number on both tasks. This allowed us to analyse how children perform on the successor Give-*N* task in relation to their cardinality knowledge as measured by the cardinality Give-*N* task. Following the most commonly used coding procedure for the Give-*N* task (see Wege et al., 2025, for a systematic review of different coding approaches that have been used in the literature), scores were based on the highest number children gave correctly at least twice, i.e. a child who correctly fed “Benny Bin” two blocks on at least two trials and failed to give 3, 4, or 6 blocks twice or more, received a score of 2. On the standard Give-*N* task, this would classify the child as a two-knower. Children who gave 6 blocks correctly at least twice on the standard Give-*N* task would classify as CP-knowers.

However, we also used the (in our view, superior) Bayesian modelling approach (Lee & Sarnecka, 2011; Negen et al., 2012) to score and compare children’s performance on both tasks. The Bayesian model calculates the likelihood of any requested number being the child’s highest known number. It considers performance across all trials of a Give-*N* task. While the “two out of three” rule would, for example, classify a child as knowing all the numbers up to 3 if they just gave 3 correctly twice, the Bayesian knower-level model would also consider whether children gave 1 and 2 correctly and whether children ever gave 1, 2 or 3 when asked for any other number. Based on the highest likelihood assigned by this model, children again received a score between 1-6 on each Give-*N* task.

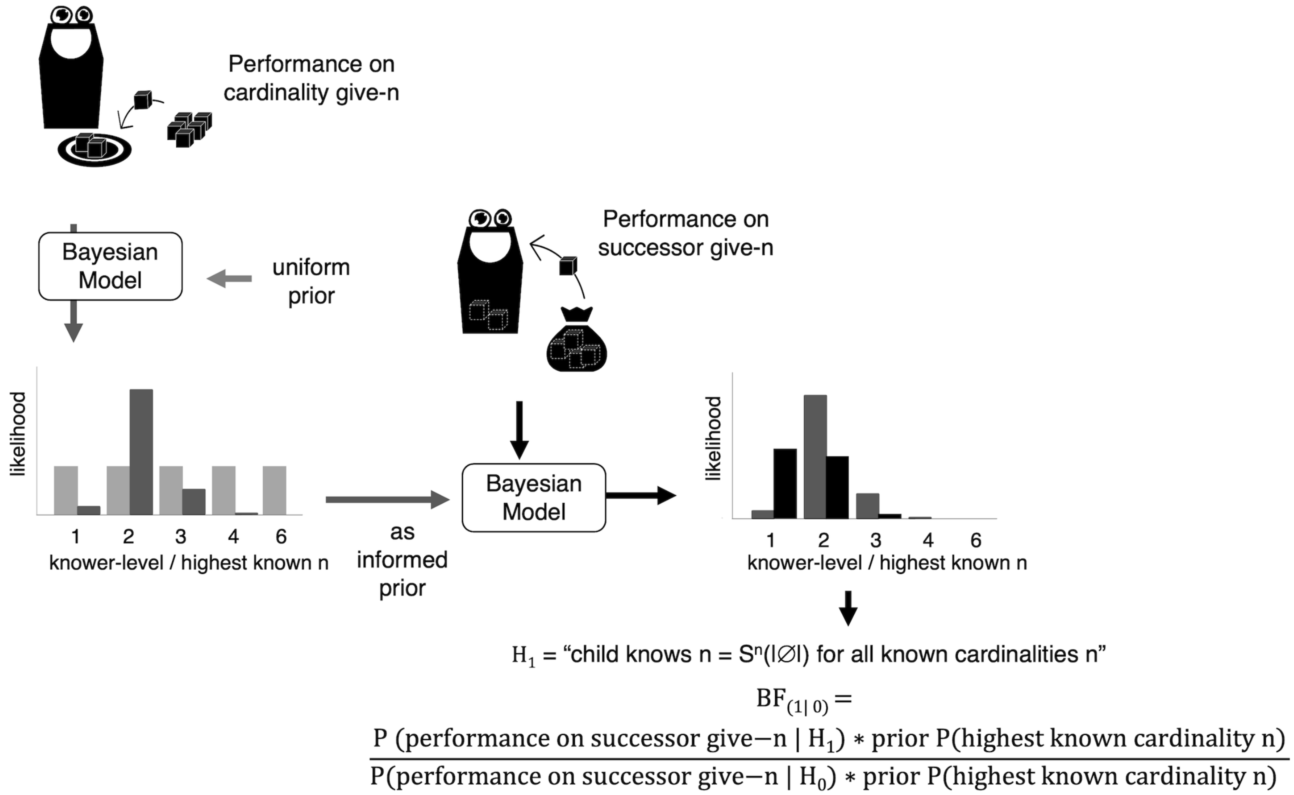
Analysis

To analyse consistency between children’s performance on both tasks, we computed crosstables and metrics of agreement. There is ongoing debate if scores on a Give-*N* task should be considered as nominal, ordinal, or continuous data (Marchand et al., 2022; Wege et al., 2025). For comparison, we computed several metrics of agreement with 95% confidence intervals using open source packages and RStudio software (Bates et al., 2015; Friendly & Meyer, 2015; RStudio Team, 2021). On a nominal scale, we computed percent agreement as the percentage of children who received the same score in both tasks and we computed unweighted Kappa coefficients. On an ordinal scale, we also computed weighted Kappa coefficients. Weighted Kappa assigned linear weights to score differences such that a two-point difference in scores is twice as severe as a one-point difference. Finally on a continuous scale, we calculated the Intra-class correlation (ICC) between scores. We concur with previous recommendations to treat scores that reflect children’s highest known number as nominal or ordinal data (Marchand et al., 2022; Wagner et al., 2019). Hence, we consider Kappa coefficients to be the most suitable metric of agreement.

In a final analysis, we used a novel Bayesian modelling approach to specifically quantify evidence for or against the hypothesis that children understand that $n = S^n(|\varnothing|)$ for all n they know the cardinality of (Figure 2). The Bayesian modelling approach allowed us to first derive the likelihood that a child knows the cardinalities up to a certain number *n* between 1 and 6 based on their performance on the standard Give-*N* task. We then used each child's likelihood distribution of known cardinalities as a prior probability when analysing their performance on the successor Give-*N* task. The resulting posterior likelihood distribution hence represents the likelihood of a child with certain probabilities of knowing the cardinalities of the numbers up to *n* to also know $n = S^n(|\varnothing|)$ up to those numbers *n*. Using this posterior distribution allowed us to calculate a Bayes Factor for each child quantifying evidence in favour of the hypothesis that children know that $n = S^n(|\varnothing|)$ (as evident in their performance on the successor Give-*N*) for all *n* they know the cardinality of (as depicted in the prior distribution of highest known *n* based on performance on the cardinality Give-*N* task). To calculate this Bayes Factor, we first determined the highest *n* children know the cardinality of as the *n* with the highest likelihood after analysing performance of the cardinality Give-*N* task with the Bayesian model. After adding performance on the successor Give-*N* task to the model, we then took the posterior probability for this *n* after and divided it by the sum of posterior likelihood for all other possible *n*.

Figure 2

Overview of the Bayesian Modelling Approach Used to Quantify Evidence for or Against the Hypothesis That Children Understand That $n = S^n(|\varnothing|)$ *for All n They Know the Cardinality of*



Note. First, children's knower-level was modelled based on their performance on the cardinality Give-*N* task and a uniform prior. The resulting posterior distribution of the child's knower-level served as a prior when modelling children's highest known *n* based on their performance on the successor Give-*N* task. The resulting posterior probabilities were used to calculate a Bayes factor quantifying evidence for or against our hypothesis.

As an example: Based on their performance on the cardinality Give-*N* task and a uniform prior, a child may have a 70% likelihood of knowing the cardinalities up to $n = 2$, but also a 23% likelihood of knowing cardinalities up to $n = 3$ and smaller likelihoods for $n = 1$, 4 and 6. This likelihood distribution serves as prior probability when considering performance on the successor Give-*N* task. Assuming the child only succeeded on one trial to give 2 on the successor Give-*N*-task, their likelihood for knowing that $n = S^n(|\varnothing|)$ for all numbers up to $n = 2$ as depicted in their posterior distribution may now be only 45%. Their likelihood for *not* knowing that $n = S^n(|\varnothing|)$ for all numbers up to $n = 2$ can be depicted as the counter-probability, 1-45% = 55%. Dividing these two probabilities would provide us with a Bayes Factor of 0.82. This Bayes Factor quantifies the evidence on the hypothesis that this child knows that $n = S^n(|\varnothing|)$ for all known cardinalities n. Bayes Factors below 0.2 were interpreted as evidence against this hypothesis and Bayes Factors above 5 were interpreted as evidence supporting the hypothesis. This study is the first to use the Bayesian model for the Give-*N* task in such a way to test hypotheses regarding children's performance on two instances of the Give-*N* task.

Results

Consistency of Highest Known Number in the Cardinality and Successor Task

Table 1 reports metrics of the consistency between scores. Firstly, we found that the "two out of three" rule and the Bayesian model produce consistent scores for the successor Give-*N* task. From this analysis we can conclude that deriving the highest known number from performance on the successor Give-*N* task is an appropriate scoring approach. The scoring assumptions of the Bayesian model apply and produce scores that depict a highest known number reliably. Secondly, we found overall very high consistency between children's scores on the cardinality Give-*N* tasks and the successor Give-*N* task. For over 70% of the children in our sample the highest known number on the successor Give-*N* task is identical to their cardinality knower-level. In fact, the 95% confidence intervals of agreement metrics between scores on both tasks overlapped with previously reported simple re-test reliability for the cardinality Give-*N* task (Agreement = 77%, $\text{Kappa}_{(\text{nominal})} = .71$; Kappa $\text{weighted}_{(\text{ordinal})} = .87$; $\text{ICC}_{(\text{continuous})} = .97$) (Marchand et al., 2022).

Table 1

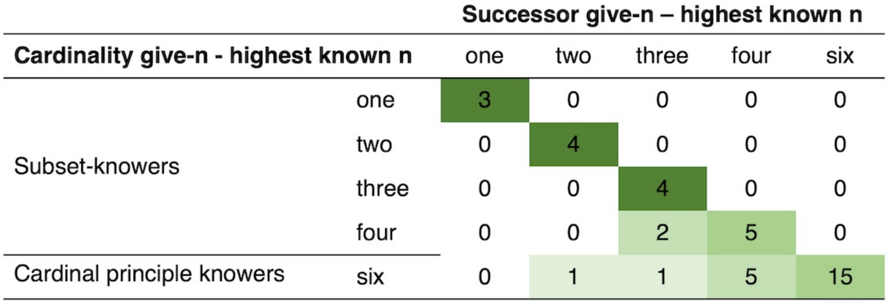
Metrics of Consistency Between Children's Scores Derived From the Two Scoring Methods and Between Scores on the Cardinality Give-N and Successor Give-N Tasks

Task version and analysis	**Cardinality Give-*N* Bayesian model**	**Successor Give-*N* "two out of three"**
Cardinality Give-*N* ("two out of three")		
Agreement	93%	77.5%
$\text{Kappa}_{(\text{nominal})}$	.885 [.76; .99]	.685 [.51; .86]
Kappa $\text{weighted}_{(\text{ordinal})}$	.945 [.88; .99]	.786 [.63; .94]
$\text{ICC}_{(\text{continuous})}$	.978 [.96; .99]	.839 [.74; .90]
Successor Give-*N* (Bayesian model)		
Agreement	75.0%	85%
$\text{Kappa}_{(\text{nominal})}$	.621 [.43; .81]	.767 [.61; .92]
Kappa $\text{weighted}_{(\text{ordinal})}$	.732 [.56; .90]	.846 [.72; .97]
$\text{ICC}_{(\text{continuous})}$	.796 [.68; .88]	.899 [.83; .94]

Figure 3A and 3B show children's scores on the cardinality Give-*N* task in reference to the successor Give-*N* task based on both the "two of out three" scoring and the Bayesian model scoring. Each child received a score between 1-6 on each task representing their highest known number *n* for this task. Most children who received a score of 6 on the cardinality task and were classed as CP-knowers also succeeded at giving numbers up to 6 on the successor Give-*N* task. Most subset-knowers, including all one-, two- and three-knowers, also succeeded up to their highest known number on the successor Give-*N* task.

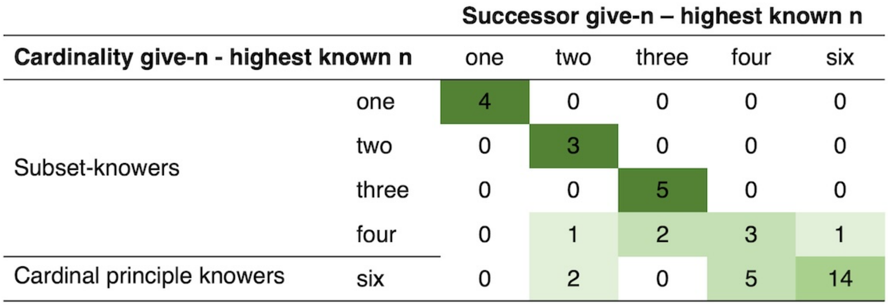
Figure 3

Children's Scores on the Cardinality Give-N Tasks in Reference to Their Scores on the Successor Give-N Task for A) the "Two out of Three" Scoring Method and B) the Bayesian Model



Cardinality give-n - highest known n		**Successor give-n – highest known n** one	two	three	four	six
Subset-knowers	one	3	0	0	0	0
	two	0	4	0	0	0
	three	0	0	4	0	0
	four	0	0	2	5	0
Cardinal principle knowers	six	0	1	1	5	15

A) "two out of three" scores



Cardinality give-n - highest known n		**Successor give-n – highest known n** one	two	three	four	six
Subset-knowers	one	4	0	0	0	0
	two	0	3	0	0	0
	three	0	0	5	0	0
	four	0	1	2	3	1
Cardinal principle knowers	six	0	2	0	5	14

B) Bayesian Model scores

Uncertainty Around Scoring Highest Known Number

We calculated scores representing children's highest known number on both tasks based on both the "two out of three" rule and based on the Bayesian model. Most children received the same scores for each task from both scoring methods. There is however some level of uncertainty associated with how well the scores on each task represent children's highest known number for cardinality and the successor function. This is largely due to the challenges of conducting standardised testing with young children, which limited our data collection to only a low number of trials on each Give-*N* task and for each number *n*. The Bayesian model allowed us to quantify the uncertainty around each child's score that resulted from having limited and likely noisy data.

Using the Bayesian model, we scored children's highest known number as the number with the highest likelihood of being the highest known number. While many children were assigned a score based on an >80% likelihood, others received a score based on just a 60-80% likelihood or even less. Considering this unique uncertainty around each child's highest known number for both for the cardinality and successor task, how strong is the evidence that any given child succeeded on the successor Give-*N* task for all *n* they know the cardinality of?

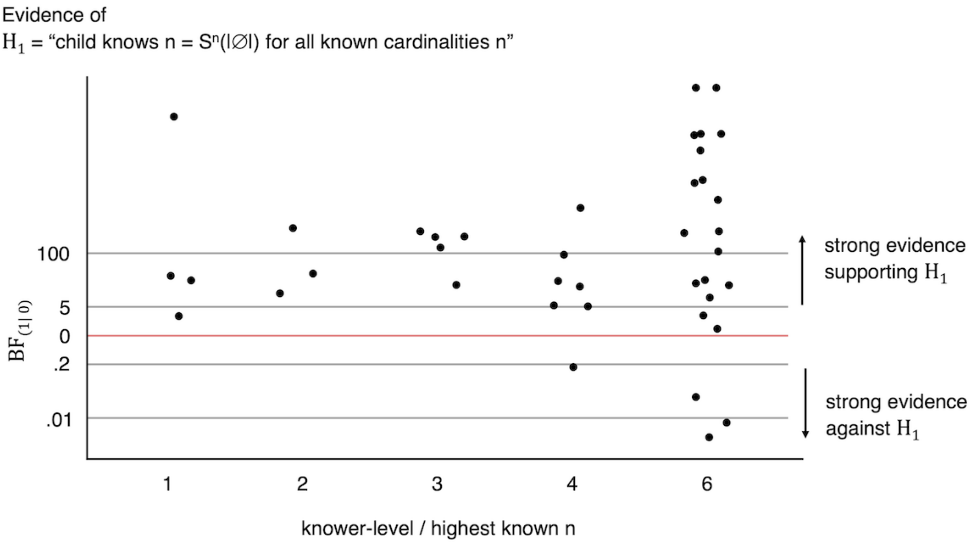
Quantifying Evidence of Each Child's Understanding That $n = S^n(|\varnothing|)$ for All Numbers n They Know the Cardinality of

Based on considerations about the uncertainty around the scores children received in both Give-*N* tasks, we developed a new analysis approach. As described above, we used the properties of the Bayesian model to address two questions: (1) is performance on the successor Give-*N* task sufficiently consistent with prior assumptions about children's knower-levels to provide strong evidence that children performed accurately up to their knower-level? And (2) for children who in our initial scoring were not assigned the same scores in both tasks, is their performance on the successor Give-*N* task sufficiently inconsistent with prior assumptions about their knower-level to provide strong evidence that they did not perform accurately up to their knower-level? In sum, we modelled the uncertainty around each child's assigned scores on the two Give-*N* tasks to quantify evidence for/against the hypothesis that the child knows that $n = S^n(|\emptyset|)$ for all their known cardinalities n.

For most children (83%) we found strong evidence ($\mathrm{BF}_{(1|0)} > 5$) that our data supports the hypothesis that the child knows that $n = S^n(|\emptyset|)$ for all known n (Figure 4). These children gave responses to all requested numbers on the successor Give-*N* task such that their response patterns supported the prior assumption about their known numbers based on their performance on the cardinality Give-*N* task. This included all subset-knowers except for one four-knower. The three other children for whom we found strong evidence against our hypothesis were CP-knowers. They all received a score of 6 on the cardinality Give-*N* task but failed to give 6 correctly on the successor Give-*N* task. They all gave numbers 1-4 correctly and received a score of 4 (in both the "two out of three" and the Bayesian scoring). Overall, we found strong evidence to support the hypothesis that subset-knowers know that $n = S^n(|\emptyset|)$ for all numbers they know the cardinality of. Our findings additionally indicate that some CP-knowers also only know this for numbers up to four and not yet for larger numbers.

Figure 4

Individual Bayes Factors Arranged by Children's Knower-Level as Evidence for the Hypothesis That the Child Knows That $n = S^n(|\emptyset|)$ *for All Known Cardinalities* n



Discussion

How do children come to understand the structure of the number system? In this study, we investigated children's early understanding of how numbers, such as "three" and "six" represent cardinalities of sets and how these cardinalities relate to one another. We used the Give-*N* task to assess children's cardinality knowledge. Children were classified based on the highest number *n* they could produce as a set of openly visible items. They were classified as subset-knowers if they could produce sets with cardinalities only up to a certain number, (e.g., three-knowers who know that one = |{a}|, two = |{a, b}| and three = |{a, b, c}|). Children who demonstrated knowledge of how counting tracks cardinality as evidenced by their ability to produce sets with cardinalities larger than four were classified as CP-knowers. It may be that only CP-knowers have made a conceptual induction about the structure of the number system (Carey, 2000, 2009). As children discover how counting tracks cardinality, they may concurrently discover the recursive meaning of the successor function: the cardinality of any number is the set created by adding one to an empty set for every count it takes to reach that number in the sequence, $n = S^n(|\{\varnothing\}|)$, such that three is not just the set $|\{a, b, c\}|$ but also $|\{a\} \cup \{b\} \cup \{c\}|$.

In the current study we therefore tested whether learning the cardinal principle coincides with learning that $n = S^n(|\varnothing|)$? And, in case it does not, whether subset-knowers already understand that $n = S^n(|\varnothing|)$ for all n they know the cardinality of, i.e. that their local understanding of the successor function is in line with their local understanding of the cardinal principle. We developed a methodology to assess children's understanding of $n = S^n(|\varnothing|)$ in relation to their cardinality knowledge. In a modified version of the Give-*N* task we assessed children's successor knowledge: the knowledge that e.g., three = $|\{a\} \cup \{b\} \cup \{c\}|$. In this task, children had to produce sets by successively adding one while the set they created remained occluded. We found that just as for the cardinality Give-*N* task, children consistently succeeded for all numbers up to a highest number on the successor Give-*N* task. Hence, the same scoring methods could be used to measure children's cardinality and successor knowledge as their highest known number on the respective Give-*N* tasks. By assessing the consistency and/or disparity in children's highest known number on the cardinality and successor Give-*N* tasks, we were able to directly relate each child's knowledge of the successor function to their cardinality knowledge.

Based on children's behaviour on the standard and modified versions of the task, we found remarkable consistency between the range of numbers children know the cardinality of and the range of numbers children know the successor function for. All but one subset-knower could produce the same cardinalities both by placing items visibly together as a set, and also by successively adding one item to an occluded set. Even considering how estimates about children's highest known number on either task are somewhat uncertain, for almost all children we found strong evidence that the highest known number on the successor Give-*N* task was identical to the highest known number on the standard Give-*N* task, i.e. their cardinality knower-level. This is with exception of a small number of CP-knowers. Although they could produce sets of six items visibly placed together they failed to produce sets of six items when the task required knowledge of the successor function $S^6(0) = 6$. Instead, for these CP-knowers knowledge of the successor function appeared to be limited at numbers up to four.

From these findings we can provide answers to our initial research questions. First, because subset-knowers already demonstrated local knowledge of the successor function and because some CP-knowers still had limited knowledge of the successor function, we can conclude that learning the cardinal principle *does not* coincide with learning that $n = S^n(|\varnothing|)$. Further, for almost all cardinal subset-knowers in our sample we found strong evidence that for any number they know the cardinality of they also know how to produce this cardinality by successively adding one to an empty set. We conclude that subset-knowers already understand that $n = S^n(|\varnothing|)$ and that they likely do so for all n they know the cardinality of (i.e. cardinal two-knowers were also successor two-knowers). In other words, children show local understanding of the successor function, as applied to numbers within their knower-level, prior to demonstrating global understanding of the successor function that can be applied to all numbers. This parallels the development of cardinal principle understanding, where subset-knowers show local understanding of the cardinal principle prior to global understanding of the cardinal principle. These findings add novel insights about how children discover the structure of the number system.

Children Learn the Successor Function Alongside the Cardinalities of the First Numbers

Our finding that children know that $n = S^n(|\varnothing|)$ for all n they know the cardinality of suggests that structural knowledge of the number system is acquired progressively. Rather than learning that the successor function is fundamental to the structure of the number system in a moment of conceptual induction, children may build this knowledge alongside learning how the first small numbers represent cardinalities of sets. This conclusion raises the question of the mechanisms by which children learn the successor function alongside learning the cardinalities of the first small numbers.

The set representations that are assumed to underlie associative mappings between the first numbers and cardinalities in the form of e.g., three = $|\{a, b, c\}|$ inherently cannot encode how cardinalities are defined by the successor function (Feigenson & Carey, 2005; Uller et al., 1999). Yet, in our study children's knowledge of cardinalities was almost perfectly consistent with their knowledge of how these cardinalities are defined by the successor function. There are two possible mechanisms to explain this. First, children's local understanding of cardinality may support local understanding of the successor function. Children may succeed in the successor Give-*N* task by updating set representations sequentially as each item is added to the set and then recognising associative mappings between the resulting set and number words (i.e. that "one" is $|\{a\}|$ and "two" is $|\{a, b\}|$ and "three" is $|\{a, b, c\}|$). Alternatively, children's local understanding of the successor function may support local understanding of cardinality through a limited version of Carey's bootstrapping hypothesis. For example, children may recognize that "two" is both the next number in the sequence after "one" and that the set associated with "two" ($|\{a\} \{b\}|$) is the set created by adding one item to the set associated with "one" ($|\{a\}|$). However, initially they do not make the conceptual leap proposed by Carey that this applies to all numbers in their count sequence, but understand it locally only related to a specific number at a time.

Our study cannot conclusively distinguish between these alternative mechanisms. All but one of the subset-knowers in our study succeeded on the successor Give-*N* for the same numbers that they succeeded on the cardinality Give-*N*. It is not clear whether this is due to our relatively small sample size or because it is rare for children to perform differently across the two tasks. Future research with larger samples may identify more children for whom there is an asymmetry in their performance across the two tasks. If these children succeed at the cardinality Give-*N* for higher numbers than the successor Give-*N* then this would suggest that local cardinality understanding supports local successor understanding, or vice-versa if they succeed at the successor Give-*N* for higher numbers than the cardinality Give-*N*. Our findings however strongly suggest that subset-knowers do have some structural knowledge of the number system that goes beyond associative mappings between numbers and cardinalities of sets.

Cardinal Principle Knowers Still Require Counting Practice to Expand Knowledge of the Successor Function

The suggestion that children discover the structure of the number system via continuous practice with counting may be further supported by our findings regarding CP-knower's knowledge that $n = S^n(|\emptyset|)$. For most CP-knowers who succeeded at giving numbers up to six on the standard Give-*N* task, we found strong evidence that they could also give numbers up to six correctly by successively adding one to an empty set. However, for three CP-knowers, we found strong evidence to the contrary. They appeared to only succeed on the successor Give-*N* task for numbers up to four but consistently failed at giving six when successively adding one to an empty set. There are two possible explanations for this finding. Both suggest that children who are CP-knowers may still require practice with the counting procedure to get a solid grip on the fact that $n = S^n(|\emptyset|)$, but they differ in regard to what exactly needs to be practiced.

The first possible explanation was offered by Schneider and colleagues (2021a). They proposed that to reason about the successor function, novice CP-knowers, just as subset-knowers, rely on associative mappings between the first small number words and cardinalities, i.e. that local cardinality understanding supports local successor understanding. Their knowledge of the structure of the number system is hence limited at four and they lack insight into the set operations that relate numbers beyond four to each other, e.g., that six is exactly one more than five which is one more than four. Given the limitations of exact set representations, children are not able to consistently use cardinality understanding to succeed on the successor Give-*N* task. To expand their knowledge of the structure of the number system to larger numbers, CP-knowers may need further practice with the exact order of the number sequence and the relation of numbers with each other (Barner, 2017; Buijsman, 2019; Rousselle & Vossius, 2021; Spaepen et al., 2018).

The second possible explanation regards practice of the counting routine itself given the cognitive limitations of young children. CP-knowers may have succeeded to give six correctly on the cardinality Give-*N* task because they could take their time to count items, re-start their counts if necessary or even use counting to self-correct the set they produced. Because such counting behaviour affects whether children produce larger sets correctly, even the distinction between four-knowers and CP-knowers on the cardinality Give-*N* tasks is not always reliable (Krajcsi, 2021; Marchand et al., 2022; Wege et al., 2025). On our successor Give-*N* task, any counting mistakes or distractions that occurred during set production could have led to an incorrect response. Indeed, we did observe such counting mistakes for one of the CP-knowers who failed to give six correctly. They counted aloud "one, two, three, four, five…five, six" giving seven items. CP-knowers may not have failed to produce larger numbers because their knowledge of $n = S^n(|\emptyset|)$ does not yet include numbers above four. Instead, they may have failed because they are not yet secure with the counting routine.

Keeping track of their counts is still cognitively very demanding and error prone (Coolen et al., 2021; Xenidou-Dervou et al., 2013), therefore these errors may represent procedural rather than conceptual limitations. If this account is correct, some CP-knowers may need further practice applying the counting routine itself.

Our data cannot distinguish between these two mechanisms. However, while the second account is a good explanation of our findings, it does not also account for CP-knowers' lacking understanding of the successor-functions observed in previous studies. While our study provided the first evidence of children's recursive understanding of the successor function ($n = S^n(|\varnothing|)$) for numbers up to six, previous studies have assessed children's subsequent understanding of the successor function ($S(n) = n + 1$), for numbers up to but also above 6 with the unit-task. The unit-task does not demand accurate counting performance but rather counting on from a given number *n*. This raises the question of how children's understanding of these two aspects of the structure of the number system relies on the same or possibly different insights. To further map out how children come to understand the structure of the number system, it would be fruitful to investigate how an understanding of $n = S^n(|\varnothing|)$ and $S(n) = n + 1$ develop in conjunction and longitudinally alongside children's continued practice with the counting routine and the number sequence.

Continued practice with the counting routine and the number sequence could make several pieces fall into place and allow children to gain a mature understanding of the structure of the number system. Practice with the counting routine and the number sequence helps children gain a better appreciation for how the numbers are ordered (Gilmore & Batchelor, 2021), how each number in the sequence has a successor that is one more and a predecessor that is one less (Sella & Lucangeli, 2020) and that the numbers in the list are infinitely generated over recursive rules (Guerrero et al., 2020).

Conclusion

Children discover aspects of the structure of the number system before they learn how counting tracks cardinality. We found that children who do not yet understand counting already understand how the cardinalities of "one" through "four" can be produced by successively adding one to a set. We conclude that as children learn the cardinalities of the first small numbers, they also learn how these cardinalities are related by the successor function. After understanding the cardinal principle of counting, children likely expand this knowledge to larger numbers and eventually all numbers with continued practice of the counting routine and the number sequence.

Funding: This research was supported by a Loughborough University postgraduate studentship awarded to Theresa Elise Wege. This work was partially supported by UKRI Economic and Social Research Council [grant number ES/W002914/1].

Acknowledgments: Special thanks goes to Michael Lee for advice on the statistical analyses. We further want to thank our colleagues at the Centre for Mathematical Cognition for their support and valuable feedback. We are very grateful to the organisers of Summer Scientist Week and the nursery staff who have given us the opportunity to conduct this research. Finally, we thank all parents and children who participated and without whom this research would not have been possible.

Competing Interests: The authors declared that there were no conflicts of interest with respect to the authorship or the publication of this article.

Ethics Statement: The study protocol was approved by the Loughborough University Ethics Review Subcommittee.

Data Availability: Materials and data are available on the Open Science Framework (see Wege et al., 2023S).

Supplementary Materials

The Supplementary Materials contain the following items (for access, see Wege et al., 2023S):

- Research data
- Materials

Index of Supplementary Materials

Wege, T. E., Inglis, M., & Gilmore, C. (2023S). *Children's early understanding of the successor function* [Research data and materials]. OSF. https://osf.io/6bf9w/

References

Barner, D. (2017). Language, procedures, and the non-perceptual origin of number word meanings. *Journal of Child Language, 44*(3), 553–590. https://doi.org/10.1017/S0305000917000058

Batchelor, S., Keeble, S., & Gilmore, C. (2015). Magnitude representations and counting skills in preschool children. *Mathematical Thinking and Learning, 17*(2–3), 116–135. https://doi.org/10.1080/10986065.2015.1016811

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software, 67*(1), 1–48.

Buijsman, S. (2019). Learning the natural numbers as a child. *Noûs, 53*(1), 3–22. https://doi.org/10.1111/nous.12219

Carey, S. (2000). The origin of concepts. *Journal of Cognition and Development, 1*(1), 37–41. https://doi.org/10.1207/S15327647JCD0101N_3

Carey, S. (2009). Where our number concepts come from. *The Journal of Philosophy, 106*(4), 220–254. https://doi.org/10.5840/jphil2009106418

Carey, S., & Barner, D. (2019). Ontogenetic origins of human integer representations. *Trends in Cognitive Sciences, 23*(10), 823–835. https://doi.org/10.1016/j.tics.2019.07.004

Cheung, P., & Le Corre, M. (2018). Parallel individuation supports numerical comparisons in preschoolers. *Journal of Numerical Cognition, 4*(2), 380–409. https://doi.org/10.5964/jnc.v4i2.110

Cheung, P., Rubenson, M., & Barner, D. (2017). To infinity and beyond: Children generalize the successor function to all possible numbers years after learning to count. *Cognitive Psychology, 92*, 22–36. https://doi.org/10.1016/j.cogpsych.2016.11.002

Chu, J., Cheung, P., Schneider, R. M., Sullivan, J., & Barner, D. (2020). Counting to infinity: Does learning the syntax of the count list predict knowledge that numbers are infinite? *Cognitive Science, 44*(8), Article e12875. https://doi.org/10.1111/cogs.12875

Clements, D. H., Sarama, J., & MacDonald, B. L. (2019). Subitizing: The neglected quantifier. In A. Norton & M. W. Alibali (Eds.), *Constructing number: Merging perspectives from psychology and mathematics education* (pp. 13-45). Springer, Cham. https://doi.org/10.1007/978-3-030-00491-0_2

Coolen, I., Merkley, R., Ansari, D., Dove, E., Dowker, A., Mills, A., Murphy, V., Von Spreckelsen, M., & Scerif, G. (2021). Domain-general and domain-specific influences on emerging numerical cognition: Contrasting uni- and bidirectional prediction models. *Cognition, 215*, Article 104816. https://doi.org/10.1016/j.cognition.2021.104816

Davidson, K., Eng, K., & Barner, D. (2012). Does learning to count involve a semantic induction? *Cognition, 123*(1), 162–173. https://doi.org/10.1016/j.cognition.2011.12.013

Feigenson, L., & Carey, S. (2005). On the limits of infants' quantification of small object arrays. *Cognition, 97*(3), 295–313. https://doi.org/10.1016/j.cognition.2004.09.010

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*(7), 307–314. https://doi.org/10.1016/j.tics.2004.05.002

Friendly, M., & Meyer, D. (2015). *Discrete data analysis with R: Visualization and modeling techniques for categorical and count data* (Vol. 120). CRC Press.

Geary, D. C., vanMarle, K., Chu, F. W., Rouder, J., Hoard, M. K., & Nugent, L. (2018). Early conceptual understanding of cardinality predicts superior school-entry number-system knowledge. *Psychological Science, 29*(2), 191–205. https://doi.org/10.1177/0956797617729817

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number.* Harvard University Press.

Gentner, D. (2010). Bootstrapping the mind: Analogical processes and symbol systems. *Cognitive Science, 34*(5), 752–775. https://doi.org/10.1111/j.1551-6709.2010.01114.x

Gilmore, C., & Batchelor, S. (2021). Verbal count sequence knowledge underpins numeral order processing in children. *Acta Psychologica, 216*, Article 103294. https://doi.org/10.1016/j.actpsy.2021.103294

Guerrero, D., Hwang, J., Boutin, B., Roeper, T., & Park, J. (2020). Is thirty-two three tens and two ones? The embedded structure of cardinal numbers. *Cognition, 203*, Article 104331. https://doi.org/10.1016/j.cognition.2020.104331

Hyde, D. C. (2011). Two systems of non-symbolic numerical cognition. *Frontiers in Human Neuroscience, 5*, Article 150. https://doi.org/10.3389/fnhum.2011.00150

Krajcsi, A. (2021). Follow-up questions influence the measured number knowledge in the Give-a-Number task. *Cognitive Development, 57*, Article 100968. https://doi.org/10.1016/j.cogdev.2020.100968

Le Corre, M. (2014). Children acquire the later-greater principle after the cardinal principle. *British Journal of Developmental Psychology, 32*(2), 163–177. https://doi.org/10.1111/bjdp.12029

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition, 105*(2), 395–438. https://doi.org/10.1016/j.cognition.2006.10.005

Lee, M. D., & Sarnecka, B. W. (2011). Number-knower levels in young children: Insights from Bayesian modeling. *Cognition, 120*(3), 391–402. https://doi.org/10.1016/j.cognition.2010.10.003

Lipton, J. S., & Spelke, E. S. (2006). Preschool children master the logic of number word meanings. *Cognition, 98*(3), B57–B66. https://doi.org/10.1016/j.cognition.2004.09.013

Marchand, E., Lovelett, J. T., Kendro, K., & Barner, D. (2022). Assessing the knower-level framework: How reliable is the Give-a-Number task? *Cognition, 222*, Article 104998. https://doi.org/10.1016/j.cognition.2021.104998

Negen, J., Sarnecka, B. W., & Lee, M. D. (2012). An Excel sheet for inferring children's number-knower levels from Give-*N* data. *Behavior Research Methods, 44*(1), 57–66. https://doi.org/10.3758/s13428-011-0134-4

Paliwal, V., & Baroody, A. J. (2020). Cardinality principle understanding: The role of focusing on the subitizing ability. *ZDM, 52*(4), 649–661. https://doi.org/10.1007/s11858-020-01150-0

Piazza, M., Fumarola, A., Chinello, A., & Melcher, D. (2011). Subitizing reflects visuo-spatial object individuation capacity. *Cognition, 121*(1), 147–153. https://doi.org/10.1016/j.cognition.2011.05.007

Rousselle, L., & Vossius, L. (2021). Acquiring the cardinal knowledge of number words: A conceptual replication. *Journal of Numerical Cognition, 7*(3), 411–434. https://doi.org/10.5964/jnc.7029

RStudio Team. (2021). *RStudio: Integrated Development Environment for R* [Computer software]. Boston, MA, USA. http://www.rstudio.com

Sarnecka, B. W. (2021). Learning to represent exact numbers. *Synthese, 198*(Suppl. 5), 1001–1018. https://doi.org/10.1007/s11229-015-0854-6

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition, 108*(3), 662–674. https://doi.org/10.1016/j.cognition.2008.05.007

Sarnecka, B. W., & Gelman, S. A. (2004). Six does not just mean a lot: Preschoolers see number words as specific. *Cognition, 92*(3), 329–352. https://doi.org/10.1016/j.cognition.2003.10.001

Sarnecka, B. W., Goldman, M. C., & Slusser, E. B. (2015). How counting leads to children's first representations of exact, large numbers. In R. Cohen Kadosh & A. Dowker (Eds.), *The Oxford handbook of numerical cognition* (pp. 291-309). Oxford University Press. https://doi.org/10.1093/oxfordhb/9780199642342.013.011

Sarnecka, B. W., & Lee, M. D. (2009). Levels of number knowledge during early childhood. *Journal of Experimental Child Psychology, 103*(3), 325–337. https://doi.org/10.1016/j.jecp.2009.02.007

Schneider, R. M., Pankonin, A., Schachner, A., & Barner, D. (2021a). Starting small: Exploring the origins of successor function knowledge. *Developmental Science, 24*(4), Article e13091. https://doi.org/10.1111/desc.13091

Schneider, R. M., Sullivan, J., Guo, K., & Barner, D. (2021b). What counts? Sources of knowledge in children's acquisition of the successor function. *Child Development, 92*(4), e476–e492. https://doi.org/10.1111/cdev.13524

Sella, F., & Lucangeli, D. (2020). The knowledge of the preceding number reveals a mature understanding of the number sequence. *Cognition, 194*, Article 104104. https://doi.org/10.1016/j.cognition.2019.104104

Sella, F., Lucangeli, D., & Zorzi, M. (2020). The interplay between spatial ordinal knowledge, linearity of number-space mapping, and arithmetic skills. *Cognitive Development, 55*, Article 100915. https://doi.org/10.1016/j.cogdev.2020.100915

Sella, F., Slusser, E., Odic, D., & Krajcsi, A. (2021). The emergence of children's natural number concepts: Current theoretical challenges. *Child Development Perspectives, 15*(4), 265–273. https://doi.org/10.1111/cdep.12428

Shusterman, A., Slusser, E., Halberda, J., & Odic, D. (2016). Acquisition of the cardinal principle coincides with improvement in approximate number system acuity in preschoolers. *PLoS One, 11*(4), Article e0153072. https://doi.org/10.1371/journal.pone.0153072

Simon, M. A., Della Volpe, D., & Velamur, A. (2023). Construction of the cardinality principle through counting: Critique and conjecture. *Mathematical Thinking and Learning, 25*(4), 465–480. https://doi.org/10.1080/10986065.2021.2012737

Spaepen, E., Gunderson, E. A., Gibson, D., Goldin-Meadow, S., & Levine, S. C. (2018). Meaning before order: Cardinal principle knowledge predicts improvement in understanding the successor principle and exact ordering. *Cognition, 180*, 59–81. https://doi.org/10.1016/j.cognition.2018.06.012

Sullivan, J., & Barner, D. (2014). Inference and association in children's early numerical estimation. *Child Development, 85*(4), 1740–1755. https://doi.org/10.1111/cdev.12211

Uller, C., Carey, S., Huntley-Fenner, G., & Klatt, L. (1999). What representations might underlie infant numerical knowledge? *Cognitive Development, 14*(1), 1–36. https://doi.org/10.1016/S0885-2014(99)80016-1

Wagner, K., Chu, J., & Barner, D. (2019). Do children's number words begin noisy? *Developmental Science, 22*(1), Article e12752. https://doi.org/10.1111/desc.12752

Wege, T. E., Bourque, T., Merkley, R., & Cheung, P. (2025). Thirty years of the Give-*N* task: A systematic review, reflections, and recommendations. *Journal of Numerical Cognition, 11*, Article e10577. https://doi.org/10.5964/jnc.10577

Wynn, K. (1990). Children's understanding of counting. *Cognition, 36*(2), 155–193. https://doi.org/10.1016/0010-0277(90)90003-3

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology, 24*(2), 220–251. https://doi.org/10.1016/0010-0285(92)90008-P

Xenidou-Dervou, I., De Smedt, B., van der Schoot, M., & van Lieshout, E. C. (2013). Individual differences in kindergarten math achievement: The integrative roles of approximation skills and working memory. *Learning and Individual Differences, 28*, 119–129. https://doi.org/10.1016/j.lindif.2013.09.012

MCLS
THE MATHEMATICAL COGNITION AND LEARNING SOCIETY

Journal of Numerical Cognition (JNC) is the official journal of the Mathematical Cognition and Learning Society (MCLS).

zpid
Leibniz-Institut für Psychologie

PsychOpen GOLD is a publishing service provided by the Leibniz Institute for Psychology (ZPID), Germany.