Journal of Numerical Cognition
jnc.psychopen.eu | 2363-8761

Research Reports

Processing the Order of Symbolic Numbers: A Reliable and Unique Predictor of Arithmetic Fluency

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Abstract

A small but growing body of evidence suggests a link between individual differences in processing the order of numerical symbols (e.g., deciding whether a set of digits is arranged in ascending/descending order or not) and arithmetic achievement. However, the reliability of behavioral correlates measuring symbolic and non-symbolic numerical order processing and their relationship to arithmetic abilities remain poorly understood. The present study aims to fill this knowledge gap by examining the behavioral correlates of numerical and non-numerical order processing and their unique associations with arithmetic fluency at two different time points within the same sample of individuals. Thirty-two right-handed adults performed three order judgment tasks consisting of symbolic numbers (i.e., digits), non-symbolic numbers (i.e., dots), and letters of the alphabet. Specifically, participants had to judge as accurately and as quickly as possible whether stimuli were ordered correctly (in ascending/descending order, e.g., 2-3-4; ●●●●-●●●-●●; B-C-D) or not (e.g., 4-5-3; ●●●●-●●●●●-●●●; D-E-C). Results of this study demonstrate that numerical order judgments are reliable measurements (i.e., high test-retest reliability), and that the observed relationship between symbolic number processing and arithmetic fluency accounts for a unique and reliable portion of variance over and above the non-symbolic number and the letter conditions. The differential association of symbolic and non-symbolic numbers with arithmetic support the view that processing the order of symbolic and non-symbolic numbers engages different cognitive mechanisms, and that the ability to process ordinal relationships of symbolic numbers is a reliable and unique predictor of arithmetic fluency.

Keywords: numerical and non-numerical order, arithmetic abilities, reverse distance effect, canonical distance effect, reliability, ordinality processing, symbolic numbers, non-symbolic numbers, arithmetic fluency

Journal of Numerical Cognition, 2017, Vol. 3(2), 288–308, doi:10.5964/jnc.v3i2.55

Received: 2016-05-31. Accepted: 2016-12-17. Published (VoR): 2017-12-22.

Handling Editors: Silke Goebel, University of York, York, United Kingdom; André Knops, Humboldt-Universität Berlin, Berlin, Germany; Hans-Christoph Nuerk, Universität Tübingen, Tübingen, Germany

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Numerical abilities are an integral and important part of modern society. Partly because of this insight, the last decades have seen a remarkable growth in interest to better understand the neurocognitive mechanisms of number processing and their relationship with arithmetic abilities. While early research associated the acquisition of arithmetic and mathematical skills with the development of domain-general knowledge (e.g., stages of logical thinking; Piaget, 1952), more recent evidence suggests domain-specific dimensions as one of the main building blocks of arithmetic competencies (Ansari, 2008; Butterworth, 2010; Feigenson, Dehaene, &

Spelke, 2004; Nieder & Dehaene, 2009). One of these domain-specific dimensions is cardinality—the ability to quantify the number of elements in a group of objects (e.g., array of dots). For number symbols (e.g., Arabic numerals), cardinality denotes the numerical quantity to which the number symbol refers (e.g., 4 apples). Studies of the cognitive mechanisms of cardinal processing have revealed important insights about how animals and humans represent the cardinal meaning of numbers (e.g., Nieder, 2005; Nieder & Dehaene, 2009; Vogel et al., 2017), how this representation changes as a function of developmental time (e.g., Ansari, 2008; Vogel & Ansari, 2012; Vogel, Goffin, & Ansari, 2015), and how individual differences relate to arithmetic and mathematical competencies (e.g., Bugden, Price, McLean, & Ansari, 2012; De Smedt, Noël, Gilmore, & Ansari, 2013; Holloway & Ansari, 2009; Vogel, Remark, & Ansari, 2015).

Another important property of numbers is ordinality, which has been a largely overlooked topic of research in the past. Ordinality denotes the relative position or rank of a number, and it relates to the knowledge that one number comes before or after another number. For example, the ordinal position of numbers allows us to instantly infer that one-thousand-and-two comes right after one-thousand-and-one. Inferences like this would be difficult to make using a purely intuitive understanding of cardinality alone. In the last years, there has been a growing number of studies which have aimed to better understand the cognitive and neural mechanisms of numerical order processing (Lyons & Beilock, 2011, 2013; Rubinsten, Dana, Lavro, & Berger, 2013; Turconi, Campbell, & Seron, 2006) and its relationship to arithmetic abilities (Kaufmann, Vogel, Starke, Kremser, & Schocke, 2009; Lyons & Beilock, 2011; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Rubinsten & Sury, 2011; Vogel, Remark, & Ansari, 2015).

The Relationship Between Numerical Order Processing and Arithmetic

While a large body of evidence has indicated a significant correlation between cardinality processing and arithmetic (e.g., De Smedt et al., 2013; De Smedt, Verschaffel, & Ghesquière, 2009; Holloway & Ansari, 2009; Vogel, Remark, & Ansari, 2015), only a few studies have investigated the relationship between behavioral measures of numerical order processing and arithmetic (e.g., Goffin & Ansari, 2016; Lyons & Ansari, 2015; Lyons & Beilock, 2011; Lyons et al., 2014; Vogel, Remark, & Ansari, 2015). These studies have demonstrated a significant positive association between individual abilities to order number symbols and the ability to calculate simple arithmetic problems such as multiplication, addition, and subtraction. For instance, Lyons and Beilock (2011) investigated the relationship between symbolic number ordering and mathematical achievement in a group of healthy university students. Participants performed a computerized arithmetic test (i.e., division, multiplication, subtraction, and addition) as well as a numerical order task with number symbols (i.e., determining whether three numerals are in left-to-right ascending order), a number comparison task with non-symbolic numbers (i.e., deciding which dot array is larger), and other control tasks such as working memory and numerical recognition. The results of this study showed a significant relationship between the performance of ordering symbolic numbers and mathematical achievement. Furthermore, the relationship between determining the larger of two dot arrays and arithmetic achievement was fully mediated by the performance of ordering symbolic numbers. This study was among the first to indicate that processing the order of numerical symbols is a strong predictor of arithmetic abilities.

Another useful metric that has been used to investigate ordinal processing of symbolic numbers is the numerical reverse distance effect. While a canonical distance effect—that is, greater response times for numbers that are close in numerical distance compared to numbers that are far in numerical distance—is typically found for cardinal judgments, distance effects have been shown to be reversed for ordinal judgments (Franklin, Jonides, & Smith, 2009; Lyons & Beilock, 2011, 2013; Turconi et al., 2006). More specifically, when participants are asked to determine whether symbolic numerals are in order (increasing or decreasing; e.g., 1-2-3; 3-2-1) or not (e.g., 1-3-2; 3-1-2), a reverse distance effect—that is, faster reaction times for consecutive in-order numbers compared to non-consecutive in-order numbers—has been observed. The reversal of the distance effect has been interpreted as an index for an order-specific mental operation that reflects a serial search mechanism, enabling faster recognition of sequentially and consecutively ordered numbers (Franklin et al., 2009; Lyons & Beilock, 2013; Lyons, Vogel, & Ansari, 2016; Turconi et al., 2006). While the study by Lyons and Beilock (2011) used overall reaction time as a dependent variable, a recent study provided first evidence of a relationship between the numerical reverse distance effect and arithmetic abilities (Goffin & Ansari, 2016). In this study, the authors collected data from a group of sixty healthy adults who were asked to perform a symbolic numerical order task, a symbolic number comparison task (i.e., which number is larger), and two arithmetic subtests of the Woodcock Jonson III Tests of Achievement (Woodcock, McGrew, & Mather, 2001). Results of this study showed that the canonical distance effect of the number comparison task and the reverse distance effect of the order task explained each a unique portion of variance in arithmetic abilities, even when controlling for additional measures such as visual-spatial working memory and inhibitory control.

Although there is increasing evidence for an association between individual differences in processing the order of numbers and arithmetic abilities, the number of studies that have investigated this question is still limited. While results of the discussed studies indicate a certain degree of reliability, especially for mean reaction times, the test-retest reliability of the behavioral correlates frequently used to assess ordinal processing as well as their associations with arithmetic abilities have not yet been investigated within the same sample.

How Specific Is the Relationship Between Numerical Order Processing and Arithmetic?

The evidence discussed so far has revealed a relationship between symbolic numerical order processing (i.e., Arabic numerals) and arithmetic abilities. In other words, the ability to order number symbols may play a crucial role in arithmetic. An interesting question that follows is: How specific is this relationship between number symbols and arithmetic? Similar to solving arithmetic problems, the processing of number symbols is culturally transmitted via formal education. This common background may explain the observed relationship between arithmetic and number symbol ordering (Merkley & Ansari, 2016). In contrast, non-symbolic numerical order processing is not commonly (or not as frequently) associated with arithmetic processing and as such it is an interesting question, whether a relationship between non-symbolic numerical order processing (e.g., dot arrays) and arithmetic exists?

A large number of studies have shown that preverbal infants (e.g., Brannon, 2002; Hyde, Boas, Blair, & Carey, 2010; Xu & Spelke, 2000), non-human primates (e.g., Boysen, Bernston, Hannan, & Cacioppo, 1996; Brannon & Terrace, 1998; Cantlon & Brannon, 2005), and other animals (Agrillo, Dadda, Serena, & Bisazza, 2009; Gabay, Leibovich, Ben-Simon, Henik, & Segev, 2013; Honig & Stewart, 1989; Krusche, Uller, & Dicke, 2010; Pepperberg, 2006) share a basic ability to discriminate non-symbolic numerical magnitudes (i.e., the cardinality of a set). This rudimentary ability of different species to discriminate non-symbolic numerical magnitudes has been taken as evidence to suggest a phylogenetic continuous and universal approximate number system (ANS) in which numbers are represented as imprecise and analog quantities (for a review see Cantlon, Platt, &

Brannon, 2009). An increasing body of evidence has also revealed a significant association between individual differences in the ANS and arithmetic competences (Feigenson, Libertus, & Halberda, 2013; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011; Lyons & Beilock, 2011), thus demonstrating a possible link between the ability to discriminate non-symbolic numerical magnitudes and the development of arithmetic abilities.

Given these findings, one would expect that individual differences in judging the order of non-symbolic numerical stimuli are also related to arithmetic abilities. To the best of our knowledge, only one study has thus far investigated individual differences in non-symbolic numerical ordering processing and its relationship to arithmetic abilities (Rubinsten & Sury, 2011). In this study, the processing of symbolic and non-symbolic numerical ordinality was contrasted between a group of adults who were diagnosed with dyscalculia—a severe learning disability in the domain of math—and a group of healthy adults. Participants were asked to decide whether symbolic and non-symbolic numbers were arranged in order or not. The behavioral results of this study revealed that adults with dyscalculia processed non-symbolic ordinal numbers differently compared to healthy adults. Specifically, participants with dyscalculia showed no significant reaction time differences when numbers were presented in-order as compared to mixed-order. While this study provides important insights about potential differences in symbolic and non-symbolic numerical ordinal processing, these findings are difficult to generalize to the entire spectrum of individual differences. As such, the question whether non-symbolic numerical order processing is related to arithmetic within healthy individuals is unknown, let alone whether such a relationship is reliable within the same sample of individuals.

The ability of humans to process ordinal relationships is not restricted to numerical information, but rather extends to other non-numerical dimensions such as the letters of the alphabet (e.g., Fias, Lammertyn, Caessens, & Orban, 2007; Franklin et al., 2009; Fulbright, Manson, Skudlarski, Lacadie, & Gore, 2003; Gevers, Reynvoet, & Fias, 2003, 2004). As such it is a feasible assumption that a relationship between arithmetic and other sequentially organized symbols, such as letters of the alphabet, may exist. Similar to numbers, non-numerical lists facilitate fast and precise recognition of whether an item is before or after another item. Furthermore, an increasing body of evidence has indicated that numerical and non-numerical order processing is organized similarly. For instance, Franklin, Jonides, and Smith (2009) explored the cognitive mechanisms underlying the ordinal processing of numbers and months of the year. In one condition, participants were instructed to indicate whether simultaneously presented numerals were in order (e.g., 12-13-14) or not (e.g., 13-14-12). In a second condition, participants were instructed to indicate whether the presented months of the year were in order (e.g., February-March-April) or not (e.g., March-April-February). Results of this study revealed a striking similarity in the way numerical and non-numerical ordered stimuli were processed. The authors not only found a reverse distance effect in both conditions, but they also showed that the reverse distance effect was specifically pronounced in trials in which numbers crossed a decade (e.g., 19-20-21) and months crossed the year boundary (e.g., December, January, February). These similarities suggest that numerical and non-numerical ordered stimuli are processed in a similar way and that ordinal judgments may be related to common ordinal mechanisms. As a consequence, the relationships between arithmetic and numerical order processing could be related to symbolic ordering more generally.

While the result above indicates common ordinal mechanisms between numerical and non-numerical sequences, some evidence also suggests differences between the two dimensions. For example, Zorzi and colleagues (2006), investigated the spatial organization of numerical and non-numerical ordered sequences in patients who suffer from a left spatial neglect following a right hemispheric stroke. These patients were asked to perform different mental bisection tasks with numbers, letters and months of the alphabet (i.e., individuals were asked to indicate the midpoint of corresponding stimuli intervals). Results of this bisection study with neglect patients revealed a very distinctive spatial pattern with which patients indicated the midpoint of these numerical and non-numerical intervals. The results of this study indicate that the spatial layout of these formats differ and do not constitute general characteristic of ordered features. Despite this evidence for representational differences between these formats, it remains a plausible option that a relationship between symbolic numerical ordinal processing and arithmetic achievement extends to other non-numerical dimensions such as letters of the alphabet. Whether non-numerical ordinal processing relates to arithmetic abilities is currently unknown, and to the best of our knowledge, no study has thus far tested the precise relationship between all the dimensions discussed above (i.e., symbolic numerical, non-symbolic numerical and symbolic non-numerical) with arithmetic fluency. Investigating the differential relationship of these dimensions with arithmetic fluency provides a good framework for testing whether symbolic numerical order processing constitutes a unique predictor of arithmetic fluency as well as the reliability of these associations.

The Current Study

In light of the discussed literature, a number of important questions remain to be answered. First, it is currently unknown whether behavioral correlates of ordinality processing are reliable measures (i.e., test-retest reliability of ordinal processing). Furthermore, while a growing number of studies have reported an association between symbolic numerical ordinal processing and arithmetic, little is currently known about the association between non-symbolic numerical and symbolic non-numerical ordinal processing and arithmetic. In addition, it is not known whether symbolic numerical ordinal processing constitutes a reliable and unique predictor of arithmetic abilities over and above these other dimensions. To address these questions, we asked a group of healthy adults to perform a symbolic numerical order task (i.e., Arabic numerals), a non-symbolic numerical order task (i.e., dot arrays), and a symbolic non-numerical order task (i.e., letters of the alphabet), as well as a paper-pencil test assessing arithmetic performance. Individuals were asked to perform all conditions at two different time points to investigate the test-retest reliability of the discussed measures and their associations with arithmetic fluency. We formulated the following hypotheses:

- **a.** We expect a reverse distance effect (i.e., faster reaction times for consecutive numbers) for in-order conditions of symbolic numbers as well as for letters of the alphabet. Furthermore, we expect a canonical distance effect for in-order trials of the non-symbolic number task. If these behavioral measures are reliable, we expect significant distance effects at both time points for all conditions.
- **b.** If behavioral measures of ordinal processing are reliable, we expect them to correlate between the first time point (T1) and the second time point (T2).
- **c.** Finally, we expect to find a reliable and unique association between symbolic numerical ordinal processing and arithmetic abilities. More specifically, symbolic numerical ordinal processing predicts arithmetic abilities and explains unique variance over and above possible associations between non-symbolic numbers and letters of the alphabet at T1 and at T2.

Methods

Participants

A total of 36 healthy right-handed adults (18 females; mean 23.5 years; range 20–33 years) were invited to participate in the present study. Four participants were excluded from the analyses due to missing data points. These participants had low accuracy rates (0-10% correct answers) in at least one of the distances in one of the conditions (i.e., symbolic numerical, non-symbolic numerical or letters), and therefore no canonical and/or reverse distance effects could be calculated. Thus, the final sample comprised 32 subjects (14 females; mean 23.5 years; range 20–33 years). Participants were recruited from undergraduate faculties of the University of Graz as well as from the surrounding community in Graz, Austria. All participants reported normal or corrected-to-normal vision and reported no history of neurological disorders. Participants gave informed consent prior to participating in the study. The study was approved by the local ethics board of the University of Graz.

Stimuli and Procedure

The test sessions took place in a standardized setting within specific test rooms at the Institute of Psychology, University of Graz, Austria. Participants performed three computerized ordinality tasks as well as a paper-pencil test to measure arithmetic fluency. To assess the test-retest reliability of all conditions and their relationship with arithmetic fluency, participants performed all conditions at two test sessions six to nine days apart.

Ordinality Tasks

Participants completed a symbolic numerical ordinality (i.e., Arabic numerals), a non-symbolic numerical ordinality (i.e., dot arrays), and a symbolic non-numerical ordinality (i.e., letters of the alphabet) task (see Figure 1). The stimuli of each condition consisted of three items that were either presented in-order (ascending or descending; e.g., 1-2-3; ●-●●-●●●; A-B-C), or in mixed-order (e.g., 1-3-2; ●-●●●-●●; A-C-B). Participants were instructed to indicate whether the presented triads were in-order (i.e., ascending or descending) or in mixed-order by pressing a corresponding button. The stimuli for the symbolic number task consisted of single-digit Arabic numerals ranging from 1 to 9. Stimuli for the non-symbolic number task consisted of three dot arrays ranging from one to nine dots per array. Dot arrays were constructed with the software *Numerus* (McCrink, 2015) and arranged in such a way that the overall surface area was either correlated (i.e., surface area increased with the number of dots), anti-correlated (i.e., surface area decreased with the number of dots) or not correlated (i.e., did not change with an increase or decrease in the number of dots) with the number of dots. This procedure ensured that the number was the most salient and reliable variable throughout the task. Importantly, participants were not instructed to count the individual dots of the dot arrays. The short presentation time of the stimuli forced them instead to approximate the number of dots and, therefore, the task tapped into non-symbolic number processing mechanisms (i.e., ANS) rather than into a linguistic symbolic number system (i.e., counting dots). The stimuli of the symbolic non-numerical ordinality condition consisted of three letters of the alphabet, ranging from the letter "A" to the letter "I". Distances between the three items in each task were kept constant in such a way that the inter-item distance was one (e.g., 1-2-3; A-B-C), two (e.g., 2-4-6; A-C-E) or three (e.g., 1-4-7; A-D-G). The inter-item distances in the mixed-order condition were constructed in such a way that the distances between items always doubled. In other words, the inter-item distance of item 6-2-4 is exactly twice the inter-item distance of item 3-1-2. This increase corresponded to the in-order condition, in which the inter-item distance between item 1-2-3 and item 2-4-6 also doubled. This procedure ensures that direct comparisons between distances in the in-order condition and distances in the mixed-order condition are possible.

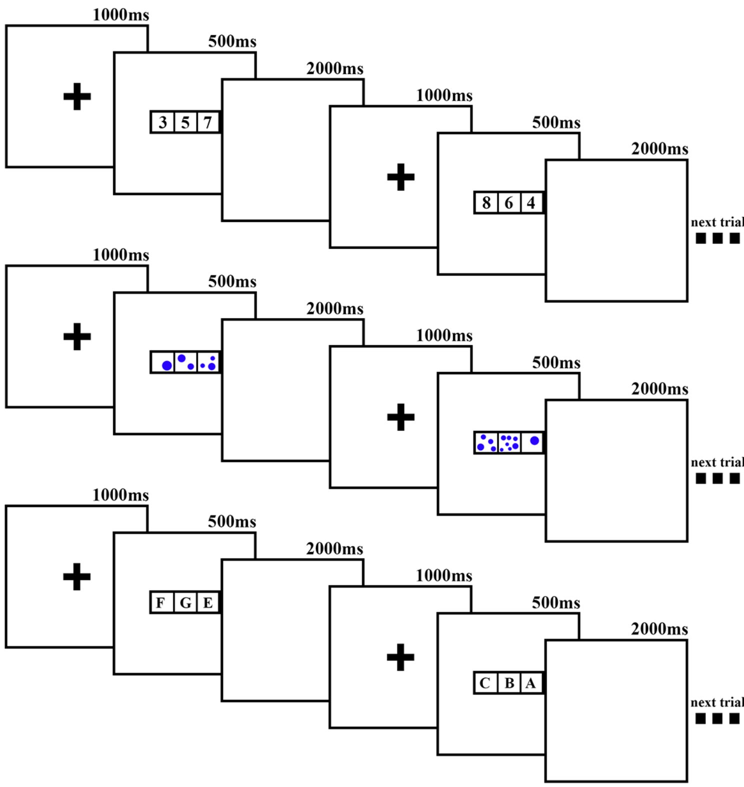

Figure 1. Examples of trial sequences for symbolic numbers, non-symbolic numbers and letters of the alphabet.

Stimuli were presented on a 14" LCD (resolution 1366 x 768) using the presentation software *PsychoPy* (v1.82.01). Each trial (see also Figure 1) started with a fixation cross that appeared for 1000ms on the screen. Stimuli triads were presented for 500ms, followed by a blank screen for 2500ms. Subjects had to respond within this time frame of 2500ms. Responses that were given after this time window were marked as incorrect. Stimuli of each condition (i.e., symbolic numbers, non-symbolic numbers, and letters) were presented in a pseudo-randomized order, containing 30 ascending and 30 descending (i.e., 60 in-order trials), as well as 60 mixed-order trials. Half of the subjects were instructed to use a left button press to indicate that stimuli were in order and to use a right button press to indicate that stimuli were in a mixed-order. The other half was instructed to respond with the reverse pattern (i.e., right button press for in-order; left button press for mixed-order). All participants were asked to use their index and middle finger of the right hand to press the corresponding button. Reaction times (RT) and accuracy (AC) rates were recorded.

Arithmetic Competence

To assess individual differences in arithmetic fluency, a new paper-pencil test based on the French Kit test of arithmetic skills (French, Ekstrom, & Price, 1963) was constructed. This test consists of multiplication, addition and subtraction problems, which participants have to solve within a given time frame. The items in the multiplication condition consist of 64 single-digit problems (e.g., 5 x 7), and 60 items in which the first factor is a double-digit number (range 10–99) and the second factor a single-digit number (e.g., 39 x 5). Items in the addition condition consist of 128 single-digit problems (e.g., 4 + 7), and of 60 items in which participants have to add 3 double-digit numbers (e.g., 30 + 98 + 59). The subtraction condition consists of 128 items in which the minuend is either a single or a double-digit (range 10-20) and the subtrahend a single-digit number (e.g., 16 - 8), and 60 items in which participants have to subtract two double-digit numbers (e.g., 82 - 31). Participants were instructed to solve as many multiplication, subtraction, and addition problems as accurately as possible within 3.5 minutes. Two versions (Version A and Version B) containing the same items in a different order were used. In the first session (T1), half of the participants were tested with Version A and the other half with Version B. Participants who were tested with Version A in T1 were tested with Version B in the second test session (T2). For the present analyses, the sum of correct answers was calculated for each participant and for each test session (i.e., T1 and T2).

Experimental Procedure

Prior to the test session, a short practice session with feedback was administered. This assured that participants understood the task, and that a correct stimulus-response assignment was established. For this, seven randomized trials were selected from the test set and presented to the participants. For each trial, feedback was given on whether the response was correct or incorrect. After this practice session, the test session started. The administration of the ordinality task was counterbalanced in such a way that one third of the participants started with the symbolic number condition (i.e., Arabic numerals), one third with the non-symbolic number condition (i.e., dot arrays), and one third with letters of the alphabet. Participants were allowed to make a short break (no longer than five minutes) between the computerized ordinality tasks. After participants finished the ordinality tasks, the arithmetic fluency test was administered. Participants performed all operations of the easy problems prior to the difficult problems. The entire test session lasted 70 minutes (individuals also participated in a second experiment—the data of this experiment are not reported in this study).

Analyses and Results

Data were analyzed using R (https://www.R-project.org, 2017). Data and R scripts for the described data analyses of the present work are available at Open Science Framework: https://osf.io/ejkp4/

Accuracy

The first analysis tested whether participants performed all conditions above chance and whether participants made less errors at T2 compared to T1 (i.e., training effect), a repeated measure analysis of variance (ANOVA) was calculated, using *Time point* (two levels: T1 and T2) and *Task* (three levels: symbolic numbers, non-symbolic numbers, and letters) as within-subject factors. Results of this analysis showed a significant main effect of *Time point* ($F(1, 31) = 100.538$, $p < .001$, $\eta^2_G = .070$) and a significant main effect of *Task* ($F(1.559, 48.334) = 42.635$, $p < .001$, $\eta^2_G = .345$). The interaction term *Time point x Task* was not significant ($F(1.426, 44.209) = 0.981$, $p = .357$, $\eta^2_G = .005$). The main effect of *Time point* is related to more errors in the first session (20% errors) as compared to the second session (14% errors). This indicates a training effect between the two test sessions in all conditions. Furthermore, post-hoc contrasts (please note that all post-hoc contrasts were corrected for multiple comparisons using False Discovery Rate (FDR), Benjamini & Hochberg, 1995) showed that participants committed more errors with letters (27% errors) than with non-symbolic numbers (18% errors)

($t(31) = -4.055$, $p_{FDR} < .001$) and symbolic numbers (6% errors) ($t(31) = -7.684$, $p_{FDR} < .001$). Participants also committed more errors with non-symbolic numbers than with symbolic numbers ($t(31) = -7.043$, $p_{FDR} < .001$). The analysis above demonstrates a training effect between T1 and T2, and that participants performed above chance level (i.e., the highest error rate was 27% errors) in all conditions. Since previous work has found associations between the reaction time and arithmetic fluency (e.g., Goffin & Ansari, 2016), no further analyses were conducted with the accuracy data.

Reliability of Reaction Time Patterns

To investigate the reliability of mean reaction time patterns for all experimental conditions, three repeated ANOVAs were calculated, using *Time point* (two levels: T1 and T2), *Order* (two levels: in-order and mixed-order), and *Distance* (three levels: Distance 1, Distance 2, and Distance 3) as within-subject factors (see Table 1 and Figure 2a).

Table 1

Analyses of Variance (ANOVA) on Reaction Time Data for Judging the Order of Symbolic Numbers, Non-Symbolic Numbers, and Letters of the Alphabet

Effects	*F*	*df*	*p*	η^2_G
Digits				
Time point	20.424	(1, 31)	.001**	.397
Order	51.933	(1, 31)	.001**	.626
Distance	33.529	(1.663, 51.539)	.001**	.520
Time point x Order	0.300	(1, 31)	.588	.010
Time point x Distance	4.240	(1.891, 58.611)	.021*	.120
Order x Distance	35.797	(1.956, 60.625)	.001**	.536
Time point x Order x Distance	3.404	(1.802, 55.873)	.045*	.099
Dots				
Time point	23.361	(1, 31)	.001**	.430
Order	16.075	(1, 31)	.001**	.341
Distance	30.053	(1.763, 54.665)	.001**	.492
Time point x Order	0.000	(1, 31)	.986	.000
Time point x Distance	0.948	(1.971, 61.093)	.393	.030
Order x Distance	1.343	(1.789, 55.462)	.269	.042
Time point x Order x Distance	0.060	(1.881, 58.317)	.933	.002
Letters				
Time point	1.822	(1, 31)	.187	.056
Order	11.602	(1, 31)	.002*	.272
Distance	15.532	(1.957, 60.671)	.001**	.334
Time point x Order	0.003	(1, 31)	.959	.000
Time point x Distance	4.663	(1.791, 55.528)	.016*	.131
Order x Distance	44.745	(1.919, 59.494)	.001**	.591
Time point x Order x Distance	0.221	(1.848, 57.296)	.785	.007

Note. *F* and *df* indicate the value and degrees of freedom for the *F* statistic, *p* is the realized significance, and η^2_G denotes generalized eta squared. Values are reported with Greenhouse-Geisser correction where necessary.
**p* < .05. ***p* < .001.

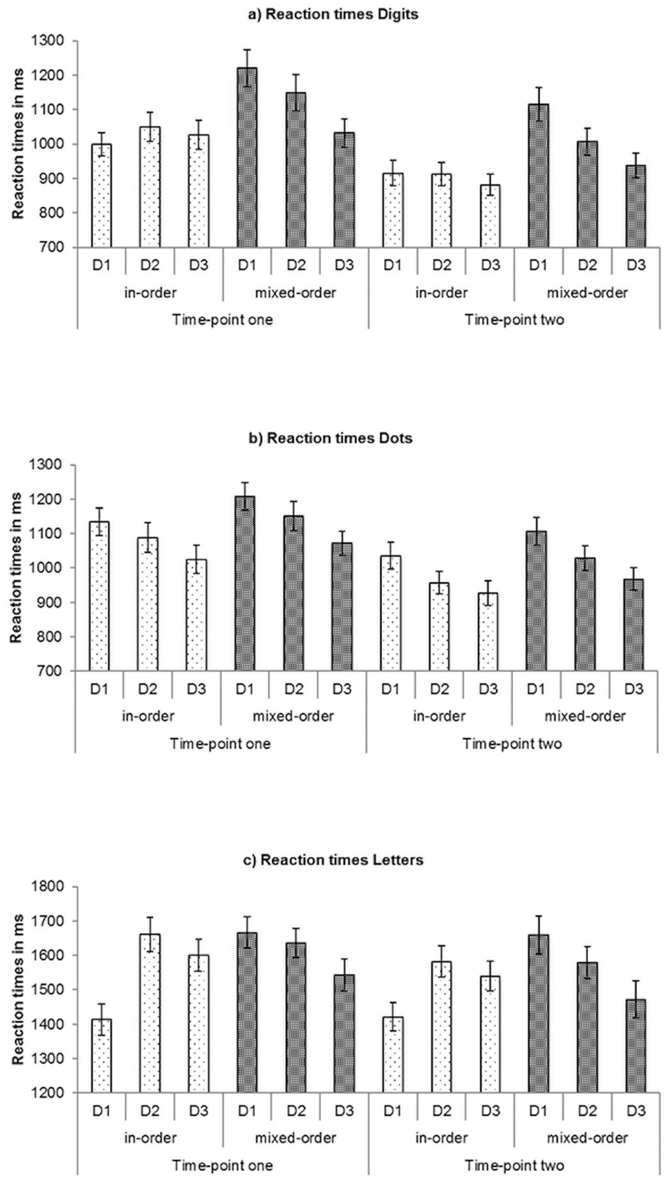

Figure 2. Bar graphs depicting reaction times of a) symbolic numbers, b) non-symbolic numbers, and c) letters of the alphabet as a function of *Time point* (T1 and T2), *Order,* and *Distance* (D1 = Distance 1; D2 = Distance 2; D3 = Distance 3).

Results for symbolic numbers showed a significant main effect *Time point* (participants were slower at T1, 1080ms, compared to T2, 962ms), *Order* (participants were faster in the in-order condition, 971ms, compared to the mixed-order condition, 1023ms), and *Distance*. Importantly, the results revealed a 3-way interaction of *Time point x Order x Distance* (Figure 2a). For in-order trials at T1, additional post-hoc analyses demonstrated that this interaction emerged from significantly faster reaction times for Distance 1 (999ms) compared to Distance 2 (1050ms; $t(31) = -2.346$, $p_{FDR} = .038$), but not compared to Distance 3 (952ms; $t(31) = -1.207$, $p_{FDR} = .258$). There was no reaction time difference between Distance 2 and Distance 3 ($t(31) = 1.273$, $p_{FDR} = .255$). For in-order trials at T2, no significant differences in reaction time between Distance 1 (915ms) compared to Distance 2 (912ms; $t(31) = 0.177$, $p_{FDR} = .861$) and compared to Distance 3 (881ms; $t(31) = 1.991$, $p_{FDR} = .074$) were found. Participants were, however, significantly slower to judge the order for Distance 2 compared to

Distance 3 ($t(31) = 2.761$, $p_{FDR} = .016$). For mixed-order trials at T1, significantly slower reaction times were found for Distance 1 (1221ms) compared to Distance 2 (1150ms; $t(31) = 3.008$; $p_{FDR} = .010$) and compared to Distance 3 (1032ms; $t(31) = 8.075$, $p_{FDR} < .001$). Participants were also significantly slower to judge the order of Distance 2 compared to Distance 3 ($t(31) = 5.606$, $p_{FDR} < .001$). The same pattern was found for mixed-order trials at T2. Reaction times were significantly slower for Distance 1 (1116ms) compared to Distance 2 (1007ms; $t(31) = 6.212$; $p_{FDR} < .001$) and compared to Distance 3 (938ms; $t(31) = 9.229$, $p_{FDR} < .001$). Participants were also significantly slower to judge the order of Distance 2 compared to Distance 3 ($t(31) = 4.969$, $p_{FDR} < .001$). The results of the ANOVA are in line with a reversal of the distance effect (i.e., faster reaction times for Distance 1 compared to Distance 2) in the in-order condition at T1, and with a canonical distance effect in mixed-order trials at T1 and T2. The absence of a significant distance effect for in-order trials at T2 indicated that reaction time in the in-order condition slightly differed as a function of whether participants performed the task at T1 or T2.

The ANOVA on reaction times of the non-symbolic number condition (see Table 1 and Figure 2b) demonstrated a significant main effect of *Time point* (participants were slower at T1, 1113ms, compared to T2, 1004ms), *Order* (participants were faster in the in-order condition, 1059ms, compared to the mixed-order condition, 1089ms) and *Distance*. Additional post-hoc *t*-tests revealed that the significant main effect of *Distance* was reflected in slower reaction times at Distance 1 (1121ms) compared to Distance 2 (1056ms; $t(31) = 4.843$, $p_{FDR} < .001$) and Distance 3 (998ms; $t(31) = 6.691$, $p_{FDR} < .001$). Participants were also slower to judge the order of Distance 2 compared to Distance 3 ($t(31) = 3.761$, $p_{FDR} < .001$). This pattern is consistent with a canonical distance effect (i.e., decreasing reaction times with an increase in distance). The absence of an *Order x Distance* and *Time point x Order x Distance* interaction indicated that distance reaction times did not significantly differ irrespective of whether dots were presented in-order or mixed-order and irrespective of whether trials were performed at T1 or at T2.

The final ANOVA performed on the reaction times of the letter condition showed a significant main effect of *Order* (participants were faster in the in-order condition, 1490ms, compared to the mixed-order condition, 1593ms) and *Distance* (Figure 2c). Furthermore, the analysis showed a significant *Order x Distance* interaction. For in-order trials, post-hoc analyses demonstrated faster reaction times at Distance 1 (1417ms) compared to Distance 2 (1622ms; $t(31) = -9.328$; $p_{FDR} < .001$) and compared to Distance 3 (1570ms; $t(31) = -6.730$; $p_{FDR} < .001$). Participants were, however, slower to judge the order of Distance 2 compared to distance 3 ($t(31) = 2.808$; $p_{FDR} = .010$). For the mixed-order condition, results showed significantly slower reaction times for Distance 1 (1663ms) compared to Distance 2 (1608ms; $t(31) = 2.092$; $p_{FDR} = .045$) and Distance 3 (1507ms; $t(31) = 7.537$; $p_{FDR} < .001$). Participants were also slower to judge the order of Distance 2 than Distance 3 ($t(31) = 3.432$; $p_{FDR} = .003$). These results are in line with a reversal of the distance effect in the in-order condition, and with a canonical distance effect in the mixed-order condition. The absence of a significant *Time point x Order x Distance* interaction demonstrated no significant differences of this pattern between T1 and T2.

Test-Retest Reliability of Individual Differences in Ordinal Processing and Arithmetic Fluency

Two different measures of ordinality, which have been used in previous studies to investigate ordinal processing (e.g., Goffin & Ansari, 2016; Lyons & Beilock, 2011), were subjected to test-retest correlation analyses: (1) overall mean reaction times and (2) the distance effects. Results of the correlation analyses on overall mean reaction times demonstrated significant and comparable correlations for all mean reaction time measures. For digits, results yielded $r = .785$, $p < .001$; for dots, $r = .791$, $p < .001$; and for letters, $r = .688$, $p < .001$.

To quantify the reliability of the distance effects, we first calculated the individual size of the distance effects for each subject by adopting the formula used in Goffin and Ansari (2016). For the number and letter condition, the reverse distance effect was calculated by:

$$DE = (meanRT_{Distance2,Distance3} - meanRT_{Distance1})/meanRT_{Distance1,Distance2,Distance3} \quad (1)$$

For the dot condition, the canonical distance effect was calculated by:

$$DE = (meanRT_{Distance1} - meanRT_{Distance3})/meanRT_{Distance1,Distance3} \quad (2)$$

Importantly, the distance effect was only calculated for in-order trials in which a reverse distance effect was found for numbers and letters. Results demonstrated significant correlations for the reverse distance effect of numbers ($r = .373$, $p < .05$) and for the canonical distance effect of dot arrays ($r = .425$, $p < .05$). However, no significant correlation was found for the reverse distance effect of the letter condition ($r = -.025$, $p = .891$).

The final test-retest analysis evaluated the reliability of the arithmetic fluency paper-pencil test used in the present study. Using a test-retest correlation analysis (Pearson's correlation), results demonstrated a significant correlation between test session one and test session two ($r = .928$, $p < .001$) of the arithmetic fluency test. Descriptive statistics of the test scores are displayed in Table 2.

Table 2

Descriptive Statistics of the Paper-Pencil Test Used to Assess Arithmetic Fluency

Measure	Minimum	Maximum	Mean	Std. Deviation
AF_{T1}	120	364	209.03	60.17
AF_{T2}	128	365	224.13	58.29

The Relationship Between Ordinality Processing and Arithmetic Fluency and Its Test-Retest Reliability

The next analysis aimed to investigate whether a reliable and unique relationship between the mean reaction times of the ordinal conditions and arithmetic fluency exists. To this end, two multiple regression analyses (method: ENTER), in which all mean reaction time variables were entered into one model, were calculated; one for T1 and one for T2. Arithmetic fluency was entered as the dependent variable and mean reaction times as independent variables. Results of the regression analyses (see Table 3) demonstrated that the mean reaction time in the number condition predicted performances in the arithmetic fluency test at T1 and T2, explaining a unique portion of variance over and above all the other variables.

The final analysis investigated whether a reliable relationship between the distance effects and arithmetic fluency exists. For this, Pearson correlations between the distance effects of all conditions and arithmetic fluency were calculated for T1 and T2. The correlation analyses revealed no significant correlations between the reverse distance effect of digits and arithmetic fluency at T1 ($r = -.219$, $p = .229$), or T2 ($r = .082$, $p = .656$). In addition, no significant correlations were found between the reverse distance effect of letters and arithmetic at T1 (r = .097, p = .597), or at T2 (r = .002, p = .991), or furthermore, between the canonical distance effect of dots and arithmetic fluency at T1 (r = .254, p = .161), or at T2 (r = .221, p = .225).

Table 3

Arithmetic Fluency of T1 and T2 Regressed on Several Measures of Numerical and Non-Numerical Ordinality

Predictor	β	*SE*	*t*	*p*	*r*
T1					
Numbers	-0.119	0.048	-2.492*	.019	-.540**
Letter	-0.033	0.056	-0.593	.558	-.159
Dots	0.012	0.053	0.228	.821	.377*
Constant	376.410	76.799			
T2					
Numbers	-0.133	0.060	-2.194*	.037	-.491**
Letters	-0.057	0.052	-1.098	.282	-.149
Dots	0.069	0.063	1.094	.283	.395*
Constant	370.922	63.628			

Note. The adjusted R^2 for T1 is 0.225, and for T2 0.202.

[a]The rightmost column r is the zero-order correlation between a given predictor and arithmetic fluency.

*p < .05. **p < .01.

Discussion

Numerical and arithmetical abilities are an important part of modern society. Recent research has demonstrated that the behavioral correlates of symbolic numerical order processing—the knowledge that one number comes before or after another number— are associated with arithmetic abilities in children and adults (Goffin & Ansari, 2016; Lyons & Ansari, 2015; Lyons & Beilock, 2011; Lyons et al., 2014). In the present study, we investigated the test-retest reliability of these behavioral correlates and whether symbolic numerical order processing (i.e., digits) constitutes a reliable and unique predictor of arithmetic abilities over and above non-symbolic numerical (i.e., dots) and symbolic non-numerical (i.e., letters) order processing.

Reliability of Ordinal Processing

Results of the analyses revealed a significant reverse distance effect for letters (i.e., faster reaction times for consecutive items compared to non-consecutive items) of the alphabet and for number symbols (i.e., digits). This observation is consistent with an increasing body of literature that has reported a reversal of the distance effect in tasks in which participants are asked to indicate the order of numerical and non-numerical symbols (Franklin et al., 2009; Lyons & Beilock, 2011, 2013; Turconi et al., 2006; Turconi, Jemel, Rossion, & Seron, 2004). While the cognitive operations underlying the reverse distance effect are not yet understood, some authors argue that the reverse distance effect is an index for an order-specific mental operation that reflects a serial search mechanism, enabling faster recognition of sequentially and consecutively ordered numbers and letters (Franklin et al., 2009; Lyons & Beilock, 2013; Lyons et al., 2016; Turconi et al., 2006).

For symbolic numbers, performed test-retest correlation analyses demonstrated that the reverse distance effect and mean reaction times are reliable measures. Both, reaction times of the reverse distance effect and mean reaction times, showed a significant correlation between the measures collected at time point one and the measures collected at time point two. Thus, demonstrating that individual differences in the behavioral correlates of symbolic numerical order processing are preserved across different time points. This result is consistent with a recent study (Goffin & Ansari, 2016) that reported a split-half reliability (correlating the reaction times of the numerical reverse distance effect between two blocks of the same session) of r = .38 for the reverse distance effect in symbolic numerical ordering. The results of the test-retest correlation analysis in the present work (r = .373) is strikingly similar to the result obtained by the study of Goffin and Ansari (2016). Together, this evidence suggests a moderate reliability of the behavioral correlates associated with symbolic numerical order processing.

In contrast to number symbols, a different test-retest pattern was found for letters of the alphabet. While a significant correlation was found for mean reaction times, no significant correlation was found for the reverse distance effect. In other words, while individual differences in mean reaction times are stable between the two time points, individual differences of the reverse distance effect of letters are not preserved across both time points. As such the test-retest reliability of the reverse distance effect in letters of the alphabet is significantly lower compared to the test-retest reliability of the reverse distance effect of number symbols. One potentiation reason for such inconsistency in the reverse distance effect of the letter condition may be that individuals are less familiar with processing the order of letters compared to processing the order of number symbols. The ordinal processing of letter sequences is less frequent in everyday life compared to the ordinal processing of numbers and, as a consequence, participants may use inconsistent strategies/cognitive mechanisms to judge the order of letters—resulting in a low reliability of the reverse distance effect in letters of the alphabet. For future work, it would be important to better understand how individuals solve numerical and non-numerical ordinal tasks. While recent research has found commonalities and differences between non-numerical and numerical ordinal processing (e.g., Di Bono & Zorzi, 2013), it is currently not well understood how these cognitive mechanisms change as a factor of experience and developmental time.

While letters demonstrated significant reverse distance effects (albeit uncorrelated) at both time points, the pooled mean reverse distance effect for number symbols was only significant at T1 but no at T2. One possible explanation for this pattern is that participants showed a training/familiarity effect between the time points, resulting in a reduction of the reverse distance effect from time point one to time point two. In other words, individuals may become more fluent in judging symbol-to-symbol associations between the two test sessions. Supporting evidence that familiarity may alter the size of the reverse distance effect comes from a study with children (Lyons & Ansari, 2015). In this study, children were asked to judge the order of single-digits (i.e., 3-4-5) and double-digits (i.e., 19-20-21). In contrast to the expectation that familiarity leads to a larger reverse distance effect (i.e., faster retrieval of consecutive items), the authors observed a relatively smaller distance effect for familiar items (single-digits) compared to unfamiliar items (double-digits). Together with the present work, these data suggest that the size of the reverse distance effect is not only altered by familiarity but becomes smaller as a factor of familiarization. As such the data suggest that the reverse distance effect constitutes a fundamental property of ordinal processing that does not arise as a function of familiarization.

In contrast to the symbolic conditions discussed above, a canonical distance effect (i.e., slower reaction times for distances that are small compared to distances that are large) was observed in the non-symbolic number condition. The canonical distance effect is in sharp contrast to the reverse distance effect observed for symbolic numbers and letters, indicating different mechanisms with which the order of non-symbolic numbers is evaluated. One interpretation of this canonical distance effect in ordinal processing is that participants engage

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in a multi-stage cardinal comparison (cardinal judgments typically reveal a canonical distance effect). At this, participants may judge whether the second dot array is larger compared to the first dot array and whether the third dot array is larger compared to the second dot array (i.e., ●<●● and ●●<●●● = ●<●●<●●●). The relative difficulty in discriminating dot arrays with a small numerical distance results in longer reaction times. As such the present data provide evidence that the ordinal representation of symbolic numbers is fundamentally different from the ordinal representation of non-symbolic number, potentially highlighting a key difference between these two numerical representations. While the underlying mechanisms between symbolic and non-symbolic ordinal judgments may differ, both numerical tasks yielded comparable test-retest reliability. Specifically, the performed test-retest analyses revealed significant correlations for the canonical distance effect as well as for mean reaction times—measures at time point one correlated significantly with the same measures at time point two. This indicates that individual differences in non-symbolic numerical order processing were preserved across time points and that individuals engaged similar mental operations.

The Relationship Between Ordinality Processing and Arithmetic Fluency

The next analyses of the present work investigated the associations between the behavioral correlates of ordinality processing and arithmetic fluency as well as the test-retest reliability of these associations. Results of the performed multiple regression analyses revealed a significant and reliable association between mean reaction times of the number symbol condition and the arithmetic fluency test, which explained a unique portion of variance over and above non-symbolic numerical and symbolic non-numerical order processing. Since multiple regression analyses control for shared variance across the included conditions, such as domain-general ordinal processes or domain-general cognitive operations, significant results can be interpreted as task specific effects that do not relate to shared processing mechanisms. This result is in line with previous findings that have found comparable results in children and adults (Goffin & Ansari, 2016; Lyons & Ansari, 2015; Lyons & Beilock, 2011; Lyons et al., 2014) and have used similar techniques to investigate the unique contribution of a task condition, over and above the entered control conditions (examples in the literature: Holloway & Ansari, 2009; Lyons & Beilock, 2011). As such the present results indicate that symbolic numerical order processing is a reliable and unique predictor of arithmetic performance.

In contrast to our expectations, the results of the present work did not reveal a significant association between non-symbolic numbers and arithmetic. Evaluating the order of non-symbolic numbers was neither a significant predictor of arithmetic achievement in the multiple regression analyses, nor did the behavioral measures of the task demonstrate a zero-order correlation with arithmetic fluency. This is an interesting finding, given that a significant body of literature has reported an association between non-symbolic number processing and arithmetic in children and adults (e.g., Feigenson, Libertus, & Halberda, 2013; Halberda, Mazzocco, & Feigenson, 2008). The existing evidence of an association between non-symbolic number processing and arithmetic is, however, largely based on cardinal judgments in which participants are asked to decide which of the two presented dot arrays is larger. There is currently not much evidence whether the processing of non-symbolic numerical order is related to arithmetic achievement or not. The absence of a significant relationship between non-symbolic numerical ordinal processing and arithmetic fluency in this study indicates that this relationship may not extent to all ordinal judgments and that the relationship between ordinal processing and arithmetic is format-specific—it is restricted to the ordinal processing of symbolic relations and does not extend to non-symbolic relations.

One explanation for these differences and for the significant and reliable association between symbolic numerical ordinal processing and arithmetic is that symbolic numerical order processing and arithmetic are culturally transmitted via formal education. More specifically, formal education provides a common framework in which knowledge about symbolic relations and their manipulations are taught. Since both operations share a similar basic understanding of these ordinal symbol-to-symbol associations, formal education may be the bridge that links the mental operations that are afforded in symbolic numerical order processing and arithmetic (see Leibovich & Ansari, 2016; Merkley & Ansari, 2016; Schneider et al., 2017 for arguments why symbolic number representations may be crucial for the development of arithmetic). In line with this argument are the results from a large cross-sectional study with 1391 Dutch children (Lyons et al., 2014). The authors of this study assessed the relationships of a number of cognitive tasks such as numerical ordering (i.e., determining whether three symbolic numbers are in numerical left-to-right order), number comparison, number line estimation, and dot array comparison, with a standardized measure of mental arithmetic ability from Grade 1 to Grade 6. The results of this study showed that the predictive value of symbolic numerical ordinal judgments for arithmetic progressively increased from Grade 1 to Grade 6. While cardinality (i.e., number comparison) was the best predictor of arithmetic performance in Grade 1 and 2, symbolic ordinal processing became the best predictor in Grade 6. This evidence indicates a shift in the relative importance of ordering abilities, from an initial weak association with arithmetic skills in younger children towards a stronger association in older children and adults. Not only does this study demonstrate the value of symbolic numerical order processing to predict arithmetic abilities, but it also indicates that the association between symbolic numerical ordinal processing may arises as a factor of education and experience.

Contrary to our expectations, the results of the present study did not show a significant relationship between the distance effects (neither the reverse distance effect nor the canonical distance effect) and arithmetic fluency. This finding is in contrast to a recent study that reported a significant association between the reverse distance effect and arithmetic abilities in adults (Goffin & Ansari, 2016). The results of this study demonstrated that both distance effects (i.e., canonical and reverse distance effect) were associated with arithmetic performance, each explaining a unique portion of variance in math ability scores. Why did the present study fail to replicate this finding? One explanation may be that different measures were used to assess arithmetic abilities. In the present work, participants were instructed to solve as many multiplication, subtraction, and addition problems as possible in the given time windows. In contrast, Goffin and Ansari (2016) defined arithmetic abilities using a composite score of the *Calculation* and *Math Fluency* subtests of the Woodcock Jonson III Tests of Achievements. While the *Math Fluency* subtest is similar to the arithmetic fluency test in the present work, the *Calculation* subtest is an untimed measure in which individuals are asked to solve mathematical problems that increase in difficulty (Woodcock et al., 2001). A better understanding of the precise relationship between the reverse distance effect and the subcomponents of arithmetic may be the key here. It would also enable a better understanding of the underlying mechanisms of the reverse distance effect and its potential association with arithmetic fluency.

Conclusion

The present work showed that the behavioral correlates of numerical order (symbolic and non-symbolic) processing are reliable measurements. Consistent with previous findings, the results also demonstrated that processing the order of number symbols is a reliable and significant predictor of arithmetic fluency, explaining a unique portion of variance over and above the mean reaction time of non-symbolic numerical and symbolic non-numerical order processing. In contrast to number symbols, non-symbolic numerical order processing was not related to arithmetic fluency whatsoever. The differential association between symbolic and non-symbolic numbers with arithmetic fluency supports the view that symbolic and non-symbolic numerical order processing engage different cognitive mechanisms, and that the ability to process ordinal relationships of number symbols is a crucial ingredient to better understand the development of numerical and arithmetic abilities.

Supplementary Materials

Data and R scripts for the described data analyses of the present work are available at Open Science Framework: https://osf.io/ejkp4/

Funding

The authors have no funding to report.

Competing Interests

The authors have declared that no competing interests exist.

Acknowledgments

The authors have no support to report.

Author Contributions

Trent Haigh, Gerrit Sommerauer, and Melanie Spindler contributed equally to the work.

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