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Empirical Research

Knowledge of Mathematical Symbols Goes Beyond Numbers

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Abstract

The written language of mathematics is dense with symbols and with conventions for combining those symbols to express mathematical ideas. For example, reading a factored polynomial function such as $f(x) = x^2(2x + 15)$ requires the knowledge that parenthesis can be used to signify function notation in one context and multiplication in another. Mathematical orthography is defined as orthographic knowledge of symbolic mathematics. It entails both knowledge of discrete mathematical symbols and the conventions for combining those symbols into expressions and equations. The ability to read text written in the base-ten system, comprised of digits and conventions for combining digits to express whole and rational quantities, is an important aspect of mathematical orthography. However, success in secondary and post-secondary programs requires more advanced mathematical orthography. The goal of this research was to determine if a simple and novel measure of mathematical orthography captures individual differences in adults' mathematical skills. Mathematical orthography was measured with a timed dichotomous symbol decision task. Adults (N = 58) discriminated between conventional and non-conventional combinations of mathematical symbols (e.g., x^2 vs. ${}^2x$; |y| vs. ||y). The mathematical symbol decision task uniquely predicted individual differences in whole-number arithmetic, fraction/algebra procedures, and word problem solving. These findings suggest that the symbol decision task is a useful index of symbol associations in mathematical development and, thus, provides a tool for understanding the role of mathematical orthography in individual differences in adults' mathematical skills.

Keywords: mathematical orthography, symbol, symbol integration, order judgment, symbol hierarchy

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Mathematics is communicated primarily through a writing system that is dense with symbols and conventions for combining symbols to express mathematical ideas. *Mathematical orthography* is the knowledge of both mathematical symbols and the conventions for combining those symbols (Headley, 2016). In early elementary school, children learn the basic elements of mathematical orthography, including digits (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9), the base-ten system for combining digits, operators ($+$, $-$, $\times$, $\div$), and relational symbols ($=$, $>$, $<$, $\geq$, $\leq$) (CCSSM; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). However, adolescents receiving algebra instruction are exposed to more advanced mathematical symbols and conventional combinations of symbols, such as superscripts (i.e., as exponents describing repeated multiplication), parentheses (which may indicate grouping in one context and a Cartesian coordinate in another), and vertical bars (i.e., indicating absolute value). Educators and mathematicians agree that learning a wide range of mathematical symbols and the conventional patterns for arranging those symbols to express mathematical concepts is critical to achievement in mathematics (Rubenstein & Thompson, 2001).

However, there are few empirical studies of mathematical orthography. In the present study, we addressed the question of whether individual differences in adults' mathematical orthography is uniquely related to their other mathematical skills.

The term *orthography* has two Greek roots: orthos and graphein. Orthos means *correct* and graphein means *to write*. Orthography, therefore, refers to stored knowledge about how to use written symbols correctly (Apel, 2011). In studies of reading acquisition, orthography is commonly investigated at the word or sub-word level with the goal of understanding how learners develop the ability to read correctly spelled words (i.e., *brain*), recognize common patterns in the language (i.e., rimes like *-est* or *-aight* and consonant blends like *sch-* or *-gn*), and detect spelling errors (i.e., *brane* or *brn*). The results of many studies suggest that orthography plays an important role in reading acquisition and may explain differences in reading skills (Apel et al., 2019; Berninger et al., 2000; Cunningham et al., 2001; Kirby et al., 2008; O'Brien et al., 2011; Wolf et al., 2000). As a theoretical construct, orthography refers to learners' *recognition* of conventional patterns that can exist and *detection* of non-conventional patterns that do not exist within a writing system (Apel, 2011). For example, readers who recognize *their*, *there*, and *they're* as correctly written and detect *thier*, *ther*, and *the're* as errors, have orthographic knowledge about written English even if they do not understand the words' meanings.

In the present research (see also Headley, 2016), we use the term *mathematical orthography* to describe knowledge about how to read or write mathematics according to the conventions for using symbolic mathematics. Mathematical orthography allows people to recognize conventional patterns and detect non-conventional patterns in symbolic mathematics. Mathematical orthography does not encompass the ability to understand the meaning of symbolic mathematics or to use the symbols effectively in more advanced ways (e.g., to comprehend a relationship or solve an equation). Notably, the development of mathematical orthography, by definition, depends on experience because it cannot emerge in the absence of exposure to the writing system for academic mathematics.

Mathematical orthography is distinct from orthography for everyday languages because the symbolic mathematics writing system has its own symbols and syntax rules (Headley, 2016). For example, these text pairs illustrate how symbolic mathematics can require readers to attend to font, relative size, relative location, and spatial orientation and develop orthography that does not apply to other texts: x^2 and x_2; $m = n$ and $m \parallel n$; x and $\times$. Well-formed expressions, equations, and relationships written in symbolic mathematics are analogous to correctly spelled words and well-constructed sentences of everyday languages in the sense that they are critical to exchanging intended meaning via text (Devlin, 2000; Skemp, 1982).

This article presents the first study that operationalizes the theoretical construct of mathematical orthography and assesses its potential to explain individual differences in mathematical performance of adults. We used a novel symbol-decision task (Headley, 2016) to measure mathematical orthography among undergraduates. The task required participants to discriminate, quickly and accurately, between conventional and non-conventional combinations of mathematical symbols (e.g., x^2 vs. ${}^2x$; $|y|$ vs. $||y$). We used correlational and regression analyses to answer the question of whether performance on the symbol-decision task explains individual differences in mathematical performance that are not accounted for by established predictors of integer arithmetic, fraction arithmetic, algebra, and word-problem solving skills. The findings from this research has implications for theories of how knowledge about the mathematical writing system relates to individual differences in mathematical skill.

Early Symbols and Arithmetic

Most learners get their first introduction to the writing system of mathematics when they are introduced to the names of digits and the base-ten system for combining digits. Knowledge of digit symbols and the development of a network of associations among those symbols is related to success in school-based mathematics (Hawes et al., 2019; Marinova et al., 2020; Merkley & Ansari, 2016; Purpura et al., 2013; Yuan et al., 2019). Cognitive scientists have identified several key relations between digit knowledge and mathematical skills. For example, cardinal knowledge of digits (i.e., Which is larger? 4 or 7; Schneider et al., 2017) is consistently correlated with arithmetic performance. Similarly, ordinal knowledge of digits (i.e., Is 2 3 4 in increasing order?; Lyons & Beilock, 2011) is also correlated with arithmetic for children (Lyons, Price, Vaessen, Blomert, & Ansari, 2014) and adults (Goffin & Ansari, 2016; Morsanyi et al., 2017; Vogel et al., 2019; Vos et al., 2017). Knowledge of digit relations – cardinal and ordinal – underlie the ability to execute and make sense of arithmetic procedures (e.g., respond to $20 - 8 = \square$ by writing 12, a smaller number than 20).

From a developmental perspective, it remains unclear how cardinal and ordinal skills support advanced mathematical skills. Vanbinst et al. (2016) argued that cardinal knowledge, specifically number comparison, is central to understanding numerical development – as important to math development as phonological awareness is to reading. However, others argue that ordinal knowledge becomes more important than cardinal knowledge over the course of mathematical development (e.g., Lyons et al., 2012, 2016). The latter view is supported by the widely replicated finding that ordinal knowledge mediates the relation between cardinal knowledge and arithmetic fluency for children as early as grade 2 (Lyons et al., 2014; Sasanguie & Vos, 2018; Xu & LeFevre, in press) and for adults (Lyons & Beilock, 2011; Morsanyi et al., 2017; Sasanguie et al., 2017; Xu et al., 2019). Current theories suggest that the relations among cardinal, ordinal, and arithmetic skills become fully integrated within a network of symbol-to-symbol associations as advanced mathematical skills emerge (Xu et al., 2019; Xu & LeFevre, in press). However, although digits and their ordering are important because they provide the structure for the base-ten system, they are a subset of the symbols and conventions required for mathematics.

Advanced Mathematics and Mathematical Orthography

Although studies about the development of various kinds of symbolic knowledge among children learning arithmetic abound, there is far less research on the role of mathematical orthography in more advanced mathematics. The *Hierarchical Symbol Integration (HSI) Model* (Xu et al., 2019) is an empirically grounded theoretical framework that describes the role of associations among symbolic digit knowledge (i.e., cardinal, ordinal, and arithmetic) and more advanced mathematical skills. In general, human development is understood as an ongoing process of change in which concepts are differentiated, reorganized, and integrated into conceptual units (e.g., Baltes et al., 2006; Case et al., 1996; Siegler & Chen, 2008; Werner, 1957). In mathematics education, acquiring skill with mathematical symbols is typically conceptualized as an ongoing process of revisiting, elaborating, and abstracting (e.g., Hiebert, 1988; Nemirovsky & Monk, 2000). The HSI model holds that, as mathematical skills increase, less advanced digit knowledge (e.g., understanding the cardinality and ordinality of digits) becomes integrated with more advanced symbol knowledge (e.g., understanding how addition and multiplication symbols describe operations). The implication of the HSI model is that, as mathematical expertise develops, learners build an interconnected network of symbolic associations in such a way that early symbol knowledge becomes less predictive of increasing mathematical skill as it is incorporated into the more advanced symbol knowledge required for more complex mathematical tasks.

The study of advanced mathematics (i.e., topics such as algebra, geometry, and calculus) involves a wide array of symbols and conventions for combining symbols. Digits and arithmetic symbols continue to play an important role. However, with the introduction of variables, the conventions for combining symbols become more sophisticated and factors such spatial location, size, and relative position become important. For example, the position and size of a digit gives different meaning to similar mathematical expressions such as $3x$ vs. x^3 (O'Halloran, 2005). The former expression requires an understanding that the $\times$ operator may be omitted when describing multiplication by a variable quantity. The latter expression is often explained to students using yet another representation of multiplication in this way: $x^3 = x \bullet x \bullet x$. Some previously learned symbols are ambiguous in advanced mathematical text. For example, $x - y$ might be read as "x minus y", -1 might be read as "negative one", and $-x$ might be read as "the inverse of x". Importantly, many symbols and conventions for combining those symbols have no analogy in arithmetic. For example, $\sqrt{0}$ and $!$ describe new operations, $\in$ and $\cup$ describe new relationships, and $|x|$ and $\tan(\theta)$ describe functions, a new concept altogether. Given the critical role symbolic mathematics plays in academic settings, determining whether individual differences in mathematical orthography exist is important to discovering its potential to support or constrain mathematics achievement.

In light of Xu et al. (2019)'s HSI model, we predicted that increasingly sophisticated mathematical orthography is related to the development of mathematical expertise in algebra and more advanced mathematical topics. For example, consider this common prompt: Expand the polynomial: $(x-3)^2 = \boxed{}$. Regardless of how learners approach the solution, the prompt demands knowledge of mathematical symbols beyond what is necessary for completing arithmetic tasks. Minimally, the learner must understand the convention that a superscript can be used outside of a parenthetic expression or adjacent to a variable. In this instance, mathematical orthography is theoretically necessary (although not sufficient) for learners to "continually evaluate the reasonableness" (CCSSM; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 8) of their results, a typical expectation in educational settings. In other words, learners need mathematical orthography to identify well-written mathematical text and distinguish it from poorly formed text. Without mathematical orthography, learners cannot detect errors in non-conventional text. For instance, a learner who does not know that exponents appear above the baseline, to the right of a base value/variable may confuse ${}^2x$ (a non-conventional combination) with $2x$ or x^2 (conventional combinations). Taking into account research in literacy education (Apel, 2011; Apel et al., 2006), we theorize that mathematical orthography develops as students engage with mathematical text through reading, writing, and speaking.

Existing Research Related to Mathematical Orthography

Research on mathematical orthography is currently limited to investigations of a single symbol and/or explorations of symbol knowledge for specific mathematical skills. For example, difficulties understanding the equal sign are well-documented among learners in elementary and middle school (Cobb, 1987; Crooks & Alibali, 2014; Rittle-Johnson & Alibali, 1999). Many children misinterpret the equal sign as an operator symbol that means "put the answer here" (Knuth et al., 2006; Powell & Fuchs, 2010). Although this interpretation is useful for completing addition problems such as $1 + 2 = \square$, children may judge equations such as 3 + 5 = 2 + 6, incorrect or meaningless (Li et al., 2008; Steinberg et al., 1991) and subsequently struggle to solve equations with missing numbers (Powell & Fuchs, 2010; Sherman & Bisanz, 2009). Problems understanding the equal sign can persist into secondary school (Chirume, 2012) .

Similarly, difficulty with minus/negative signs can interfere with skill development (Jiang, Cooper, & Alibali, 2014). Vlassis (2008) found that middle-school students had difficulty making integer calculations when procedures involved multiple minus/negative symbols (e.g., $5-(-1)=6$). Students also have difficulty solving algebraic equations when minus/negative symbols precede a variable (Herscovics & Linchevski, 1994). For example, a student solving $12-x=7$ understood how to subtract the same quantity from both sides of the equation; however, they did not know that a minus/negative symbol could precede a variable, wrote $x=7-12$ and thus produced an invalid solution (Vlassis, 2008). Notably, because the students in these studies demonstrated conceptual understanding of the negative sign, the authors attributed their errors to misunderstanding the conventions for writing equations involving negative symbols (Herscovics & Linchevski, 1994; Vlassis, 2008). The findings on single symbol misconceptions (i.e., equal and negative signs) suggest that research on mathematical orthography may provide insights into how symbol knowledge contributes to mathematical skill.

Development of mathematical orthography occurs gradually as students have more experience in mathematics and related disciplines. One consequence of these experiences is that learners develop mental representations of familiar combinations of symbols. Some researchers have shown that one cause of American students' difficulty with the equals sign is that they rarely see it used in contexts other than the canonical "equation = answer" scenario (e.g., 3 + 3 = 6; McNeil et al., 2006; Powell, 2015). In contrast, students in other cultures are exposed to other, equally correct combinations (e.g., 6 = 3 + 3), as frequently as they see the canonical pattern (Capraro et al., 2007; Li et al., 2008). Thus, familiarity with correct versions and combinations of symbols is largely a consequence of experience.

We are aware of only one study investigating students' mathematical orthography and combinations of symbols used in advanced mathematics. Headley (2016) found that, for students in grades 7 and 8, familiarity with symbol conventions was related to performance on a standardized curriculum-based math assessment. That is, students who better discriminated between conventional and non-conventional mathematical symbols (e.g., x^2 vs. ${}^2x$; $|y|$ vs. $||y)$ scored higher on a general math assessment than students who were less skilled at discriminating conventional mathematical symbols. This finding suggests that studies of mathematical orthography may deepen our understanding of how knowledge about symbols contributes to the ability to do mathematics.

Mathematical Orthography and Working Memory

Most cognitive tasks, from reading to spatial processing to mathematics, require working memory (Baddeley, 2012; Engle, 2002; Oberauer, 2019). Working memory is a domain-general cognitive process because it is associated with many aspects of cognition. Domain-specific processes by contrast, are associated with primarily with one aspect of cognition, such as mathematical cognition. To evaluate the specific influence of orthography on mathematical skills, we controlled for domain-general working memory processes as described by Baddeley's multi-component model (2001, 2012). This model has been particularly useful for understanding the relations between working memory and mathematics (DeStefano & LeFevre, 2004; Peng et al., 2016; Raghubar et al., 2010). According to the multi-component model, working memory is a limited capacity system with (a) a visuospatial sketchpad that processes visual and spatial information, (b) a phonological loop that processes articulated information, (c) an episodic buffer that links working memory to long-term memory, and (d) a central executive that coordinates and directs attention to relevant information.

Working memory processes are likely involved in the task we designed to operationalize mathematical orthography. The visual-spatial sketchpad may be necessary to notice spatial features of text such as the size and position of 2 in x^2 versus $2x$. The phonological loop may be used to mentally articulate labels such as "x squared." The episodic buffer may be used to store representations of familiar mathematical symbols. Finally, the central executive may be used to direct attention to the most relevant information. Controlling for multiple aspects of working memory using measures that have been used extensively in studies of children and adults (Peng et al., 2016; Raghubar et al., 2010) is necessary to use our task to explore individual differences mathematical cognition.

Mathematical Orthography in a Developmental Framework

Research shows that mathematical skills develop hierarchically, with lower-level skills acting as a building block for higher level skills (e.g., Cirino, Tolar, Fuchs, & Huston-Warren, 2016). Similarly, the HSI model suggests that digit symbol knowledge develops hierarchically, with lower-level knowledge becoming integrated into higher level knowledge (Xu et al., 2019). However, there are many open questions about how these hierarchies are constructed over the course of development. For example, whole number division is a building block for fraction knowledge which is, in turn, a building block for working with algebraic rational expressions (Bailey et al., 2012; Siegler et al., 2012). Accordingly, based on the HSI model, we predict that, although digit knowledge supports whole number arithmetic performance, it may become so integrated into an advanced degree of mathematical orthography that it will mediate correlations between digit skills and performance on algebraic rational expressions. In statistical terms, a correlation between basic indices of digit knowledge (i.e., cardinal, ordinal, and arithmetic) and performance on more advanced mathematical tasks would follow from HSI. However, the correlations between mathematical orthography and performance on advanced mathematical tasks should be stronger. Accordingly, accounting for mathematical orthography should reduce or eliminate the predictive power of digit tasks with respect to more advanced mathematical skills.

The Present Research

The purpose of this study was to test how adults' familiarity with the conventions of symbolic mathematics relates to individual differences in mathematical performance. We used a mathematical symbol-decision task (SDT-Math) to index individual differences in mathematical orthography (Headley, 2016). In the SDT-Math task, participants view strings of familiar mathematical symbols and decide whether the strings convey meaning in a conventional way. For example, $x - x_1$ is conventional whereas $x - {}_1x$ is not. We assume that the process of deciding if a string of symbols is conventional entails access to and use of stored knowledge about how mathematical symbols can be combined. This way of testing whether participants have mathematical orthography is simple and direct, similar to the way that lexical knowledge of written words is assessed in lexical decision tasks (Balota & Chumbley, 1984).

To control for participants' knowledge of orthographic conventions outside the domain of mathematics we designed an analogous symbol decision task using English punctuation symbols and conventions (i.e., SDT-Punctuation). In the SDT-Punctuation task, participants judged the validity of combinations of symbols that appear in written English (e.g., haven't versus haven"t, semi-annual versus semi:annual). There was no mathematical content in these strings. Because both symbol-decision tasks involve judging the validity of a complex combination of visual symbols, they may share task-specific attributes. However, given cross-linguistic research suggesting that orthographic skill is specific to the symbols and conventions of particular writing systems (Koda, 2007), we anticipated that the SDT-Math and SDT-Punctuation would tap into different stored knowledge and generate evidence of orthographic domain specificity.

To examine how adults' mathematical orthography was related to their mathematical performance we used both frequentist hierarchical linear regression and Bayesian regression. Combining the use of Bayes factors with traditional hierarchical linear regression provides distinct advantages for analysing our data (Rouder & Morey, 2012; Wagenmakers, 2007; Wagenmakers et al., 2018). First, because the Bayesian analysis provides an odds ratio, we can interpret the level of support for a proposed model (e.g., the combination of working memory, digit knowledge, and mathematical orthography) compared to an alternate model (e.g., working memory and digit knowledge; Jarosz & Wiley, 2014; Wagenmakers et al., 2018). Second, we can interpret the Bayes factors and, thus, draw conclusions about the relative importance of the variables of interest. Third, because Bayes factors are calibrated differently than *p* values, Bayes factors can provide converging evidence to support significance testing with small sample sizes. Finally, using Bayes regression, we can compare models with all combinations of predictors and, thus, draw conclusions about the variables that consistently predict the mathematical outcomes (Rouder & Morey, 2012).

In the present research, university students completed a demographic survey, cognitive control measures (i.e., working memory), measures of basic digit knowledge (i.e., cardinal, ordinal, and arithmetic), several mathematical outcomes, and the symbol-decision tasks for mathematics and punctuation. To assess whether mathematical orthography builds on basic digit knowledge and is integrated into a broad network of symbol associations, we examined the relations amongst digit knowledge, the SDT-Math, and mathematical outcomes. Using both hierarchical regressions and Bayesian regressions, we tested the relations between the SDT-Math, digit knowledge, and more advanced mathematical performance. On the assumption that orthographic knowledge of mathematical symbols is more than digits, we hypothesized that, after controlling for working memory (i.e., spatial span, backward digit span) and basic digit knowledge (i.e., digit comparison, order judgments, arithmetic fluency), performance on the SDT-Math would explain unique variance in advanced math outcomes.

In summary, the present study builds on the view that symbol associations amongst digits are foundational for whole-number arithmetic but that knowledge of the orthographic relations amongst complex symbols will predict performance on advanced mathematical tasks that incorporate those symbols.

Method

Participants

Fifty-eight participants (M_{age} = 20.0 years; Range = 16 to 34; 34 male) were recruited from students taking first- or second-year Psychology or first-year Cognitive Science courses. Students received 1.5% course credit for their participation. The sample included students from faculties of Arts and Social Sciences, Public Affairs, or Business (*n* = 27) and students from Science and Engineering who were studying STEM subjects (i.e., science, technology, engineering, or mathematics; *n* = 31). Participants provided written informed consent. All spoke fluent English. The majority (74%; *n* = 43) had attended Canadian high schools, 5% (*n* = 3) had attended US schools, 3% (*n* = 2) had attended Australian high schools and the remainder had attended schools in the

Middle East (8%, n = 5), Africa (3%, n = 2), or Asia (5%, n = 3). This study was approved by the Carleton University Research Ethics Board.

Procedure

Participants were tested individually in a quiet room. The full test battery and task order is shown in Appendix A. Testing order was fixed, except that the order in which participants completed the SDT-Math and SDT-Punctuation tasks was counterbalanced across participants.

Materials

Experimental Tasks

Mathematical orthography — The Symbol-decision Task - Math (SDT-Math; Headley, 2016) measures speeded recognition of conventional combinations of mathematical symbols. The stimuli were developed based on the curriculum expectations in the U.S. Common Core State Standards for Mathematics (CCSS-M; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Stimuli included symbols such as exponents (x^2), absolute value signs (|*y*|), subscripts ($x - x_1$), square root ($\sqrt{x}$), and fraction notation ($^x/_y$). The thirty conventional stimuli were each paired with a non-conventional item. To create the non-conventional items, the conventional symbols were either re-ordered (e.g., x^2 was transformed to ^2x), spatially re-oriented (e.g., *a* · *b* was transformed to *a* . *b)* or replaced with a visually confusing symbol with no meaning in a secondary academic context (e.g., *l* × *w* was transformed to *l* ⊗ *w*). The full set of stimuli is shown in Appendix B with characteristic indicators describing how each pair is matched for distinct ink marks and how distinct pairs varied in terms of orthography and curricular relevance (see Headley, 2016 for further explanation of item development).

The task was administered on an iPad using a specially developed application (https://carleton.ca/cacr/math-lab/apps/sdt-app/). The app was developed using X code software. On each trial, a fixation point (*) appeared at the center of the screen for 500ms followed by the mathematical symbol stimulus which remained on the screen for 1500ms. A green checkmark and a red X were shown on the screen below the stimulus. Participants were instructed to quickly and accurately decide if the stimulus was conventional (i.e., “readable”) or non-conventional. The stimuli, timing, and instructions for the task are designed to require participants to respond to visual features of the text without engaging in mathematical reasoning or mental manipulation of symbols. The check mark and X remained on the screen until the participant responded. The response and the response time were recorded for each trial. Item order was randomized. To ensure participants understood the nature of the SDT-Math task, they first completed ten practice trials using a conventional lexical decision task (e.g., items were: plain, plin, skit, skti, robe, vobe, bin, bni, gene, and gean).

Spearman-Brown split-half calculations were used to assess the reliability of the SDT-Math for two reasons. First, it is a dichotomous choice task. Second, the SDT-Math is presumed to be multidimensional because it assesses a range of symbolic mathematics conventions from various curriculum topics. Spearman-Brown split-half reliability is preferred, over Cronbach’s alpha, for measurement tools with dichotomous options (Cortina, 1993) and multi-dimensional measurement tools (Field, 2013). The scale split-half reliability for accuracy on all 60 mathematical items was strong (Spearman-Brown coefficient r_{sh} = .84). The split-half reliability was better for conventional (r_{sh} = .92) than for non-conventional items (r_{sh} = .76). As expected, Cronbach’s alpha values were weak for both the conventional (α = 0.58) and non-conventional (α = 0.57) scale items because the item set is multi-dimensional, and the response options are dichotomous.

Sensitivity scoring (*d'* and response bias *c*) was used to index performance on the SDT. In two-alternative forced choice tasks, *d'* is often used as the index of performance because it corrects for guessing by considering both hits (i.e., correct identifications) and false alarms (i.e., incorrect identifications). That is, *d'* captures the strength of response to the signal (i.e., recognizing the conventional stimuli) relative to the noise (i.e., mistaking non-conventional stimuli as conventional). Because the symbol-decision task involves stimuli that university students in non-STEM fields may only see infrequently (e.g., $f(x)$ and x^{m+n}), accuracy was the main dependent variable and sensitivity scoring was used (Diependaele et al., 2012). The *d'* sensitivity score was calculated as the difference between the standardized proportion of hits and false alarms. Proportions were standardized to a normal probability distribution with a mean of 0 and a standard deviation of 1. The formula used in SPSS to calculate was *d' = idf.normal (pHit, 0,1) – idf.normal (pFalseAlarm, 0,1)* where *p* is proportion. A high *d'* score indicates the participant is proficient at discriminating between conventional and non-conventional stimuli. For example, a score of 4.66 indicates a 99% hit rate and 1% false alarm rate whereas a *d*' of 0 indicates that there are no differences between hits and false alarms (Keating, 2005). Response bias (*c*), the tendency to either accept or reject stimuli regardless of their conventionality, was calculated as the sum of the standardized proportion of hits and false alarms divided by negative 2 (Stanislaw & Todorov, 1999) in SPSS:

*c = -0.5 * (idf.normal (pHit, 0,1) + idf.normal (pFalseAlarm, 0,1)).*

A negative *c* value indicates a bias to judging both types of symbols as acceptable whereas a positive *c* value indicates a bias to judge both types of symbols as unacceptable.

Punctuation orthography — The Symbol-decision Task – Punctuation (SDT-Punctuation) was developed as a control measure. Specifically, the SDT-Punctuation was designed to control for orthographic knowledge outside the domain of mathematics. It was administered in the same format as the SDT-Math task but was designed to assess speeded recognition of conventional punctuation. Thirty conventional stimuli were developed using ten familiar punctuation symbols such as apostrophes, commas, periods and dashes. Importantly, punctuation symbols that were common in math and/or computer programming were avoided; instead, the focus was on punctuation in written English prose.

The thirty conventional items were paired with non-conventional stimuli that were developed to mimic the transformation protocol used by Headley (2016) in developing the SDT-Math. Conventional stimuli were transformed in one of three ways to create the non-conventional stimuli: (a) character substitution; the conventional symbol was replaced with an incorrect symbol (e.g., semi-annual, semi:annual), (b) symbol order; the conventional symbol was placed in a different order than would normally be seen (e.g., don't, do'nt), and (c) spatial orientation; conventional symbols were rotated or re-sized (e.g., “Okay”,, ,Okay,,). The full set of stimuli is shown in Appendix C. Stimulus order was randomized. The stimuli were presented in the same format as the SDT-Math and the test score was calculated the same way (*d'* and response bias *c*). As with the SDT-Math, Spearman-Brown split-half reliability was the best option for assessing reliability (Cortina, 1993). Split-half scale reliability for accuracy was acceptable for conventional items (r_{sh} = 0.70) and for non-conventional items (r_{sh} = 0.67). Cronbach's alpha values respectively were 0.57 and 0.63.

Cardinal knowledge — Digit comparison was tested using the iPad app *Bigger Number* (https://carleton.ca/cacr/math-lab/apps/bigger-number-app/). This version of digit comparison was used in studies with children (Xu & LeFevre, in press) and adults (Xu et al., 2019). Participants observe a screen displaying two single digits side-by-side. They are instructed to quickly and accurately tap the digit that represents the greater quantity. The mathematical distance between the two digits was manipulated. Half the trials had a small distance (the difference ranged from 1 to 3) and half the trials had a large distance (the difference ranged from 4 to 7) (Bugden & Ansari, 2011). There were a total of 26 trials and inverse accuracy scoring was used (mean response time/mean accuracy; Sasanguie et al., 2017). Reliability was calculated based on response times for all 26 trials (Cronbach's α = .86).

Ordinal knowledge — The order judgment task is a timed pencil/paper task where participants indicate whether a series of three digits are in numerical order; ordered sequences could be either ascending (i.e., 2 4 5) or descending (i.e., 5 4 2). This version of the task was used in other research with adults (Xu et al., 2019). The digit sequences include an equal number of ordered and unordered counting sequences (e.g.., 2 3 4; 9 7 8) and of ordered and unordered neutral sequences (e.g., 1 4 7, 4 1 7). The test is two pages, with a total of 64 sequences (eight examples of four different types of sequences per page). The researcher first demonstrated the task with three practice sequences. The participant then completed six practice items, received feedback, and subsequently completed the two test pages. Time to complete each page was recorded. Scoring was the inverse accuracy (i.e., total time in seconds divided by the number correct). Reliability was calculated by comparing scores on the two pages (Cronbach's α = 0.92).

Mathematical Outcome Measures

Arithmetic fluency — The Calculation Fluency Test (CFT; LeFevre, Dunbar, & Sowinski, 2020) is a timed, pencil-and-paper test that measures fluency in multi-digit arithmetic (addition, subtraction and multiplication, Sowinski et al., 2014). The test consists of one page each of double-digit addition (e.g., 28 + 13), double-digit subtraction (e.g., 89 – 60) and double- by single-digit multiplication (e.g., 16 × 8). Participants are given one minute per page to solve as many items as possible. The total score is the sum of all the correct responses. Reliability was calculated using total correct on the three pages, Cronbach's α = 0.89.

Fraction/algebra procedures — The Brief Math Assessment-3, developed by Steiner and Ashcraft (2012), is an untimed pencil-and-paper task based on the Wide Range Achievement Test: Third Edition (WRAT 3, 1993). Questions include arithmetic with whole numbers, fraction arithmetic (e.g., 3 1⁄2 + 2 1⁄2), and algebra (e.g., 5j – w = 18, 4j – w = 24 solve for j and w). In this expanded version, we included three additional questions (also based on the WRAT 3) to increase the number of algebra and fraction problems. The revised version has a total of 13 questions. To differentiate scores on this task from whole-number arithmetic, the first three questions, which involved whole number addition, subtraction, and multiplication, were not included in the final score. Reliability on the final 10 items was similar to reliability based on all 13 items (Cronbach's α = 0.70 and 0.75, respectively).

Number line accuracy — In the iPad app *EstimationLine* (Hume & Hume, 2014a) participants judged the location of written numbers on a number line using labeled endpoints, 0 and 7000, as a reference. There were 14 trials with target numbers below the midpoint (< 3500), 14 trials with target numbers above the midpoint (> 3500) and one midpoint trial (i.e., 3500) for a total of 29 trials. The order of the trials was determined randomly for each participant. Each trial was scored as the percent absolute error (PAE) between the target number and the participant's placement of the number on the number line. The mean PAE was calculated across all 29 trials for each participant. Reliability was based on the PAEs across trials (Cronbach's α = 0.92).

Word problem solving — Participants completed the problem-solving subtest from the KeyMath Numeration test (Connolly, 2000). Participants listened to 16 story problems and provided a verbal response which was recorded by the experimenter. Participants were not given pencil and paper and thus relied on mental math to solve the problems. The total time allowed to complete the 16 items was 7 minutes. This was sufficient time for most participants to attempt all 16 problems (i.e., the mean number of problems answered was 15.8). Thus, this was not a speeded test. Scoring was total correct. Reliability was based on the 16 items (Cronbach's α = 0.77).

Working Memory Measures

To examine the relations among math-specific variables independent of the effect of working memory we included two working memory control variables.

Backwards digit span — In this task, participants hear a series of numbers and then recite the numbers in reverse order. For example, for the sequence "9, 2, 7" the correct response is "7, 2, 9". Testing began with 3 trials of a 2-digit span. The span-length increased by 1 digit after each set of 3 trials, up to a 6-digit span. Testing was discontinued when all three trials in a given span-length were incorrect. The score was the total number of correct trials. Reliability was based on the sub-scores of the first, second, and third trials of all span lengths (Cronbach's α = 0.62).

Spatial span — *PathSpan* (Hume & Hume, 2014b) is an iPad spatial span task where nine green circles arranged in a quasi-random configuration are displayed on a screen. After the participant presses the start key, a given number of circles light up briefly one-by-one. The participant then taps the circles in the same order in which they lit up. After a practice trial with a sequence length of 2, participants completed 2 trials for each span length (i.e., from 2 to 9). Testing was discontinued after both sequences in a given span were reproduced incorrectly. Scoring was the total number of correctly reproduced sequences. Reliability was based on the sub-scores of the first and second trials of all span lengths (Cronbach's α = 0.85).

Results

First, we analyzed performance on the two symbol-decision tasks (i.e., SDT-Math and SDT-Punctuation). Second, we examined the relations between these tasks and the mathematical outcome measures. Third, we explored the relations between the SDT-Math and the other mathematical measures. Fourth, we tested OLS multiple regression models for each mathematical outcome: arithmetic fluency, fraction/algebra procedures, word problem solving, and number-line estimation. Working memory measures were included in the regressions to control for general cognitive skills. Bayesian linear regressions were also used to provide converging evidence in support of the hierarchical regression models. These regressions were done using Jamovi software that contrasted the best-fitting model with a series of models that included all possible combinations of the other predictor variables. The model with the best evidence (most support) was chosen by examining the Bayes factor, $BF_{M,}$ values of all possible models. We also report the inclusive Bayes Factor, BF_{inc}. The inclusive Bayes Factor compares the probabilities of two classes of models – models that include the variable of interest versus models that exclude the variable of interest. The BF_{inc} ratio is the odds in favor of including the variable of interest. To interpret the strength of the evidence for all Bayesian analyses, we used the Jeffrey's (1961) guidelines (as cited in Table 4; Jarosz & Wiley, 2014). Specifically, evidence for the alterative hypothesis is consider anecdotal if the *BF* = 1-3, substantial if the *BF* = 3-10, strong if the *BF* = 10-30, very strong if the *BF* = 30-100 and decisive if the BF > 100.

Task Performance

Orthography Symbol-Decision Tasks

Performance on the symbol-decision tasks is shown in Table 1. Proportion of correct responses and response times were analyzed in separate 2(task: math, punctuation) x 2 (stimulus type: conventional, non-conventional) repeated measures ANOVAs.

Table 1

Mean Accuracy (Proportion Correct) and Mean Response Time (RT) Per Item for the Symbol-Decision Tasks

	Conventional		Non-conventional		Sensitivity score[b] (*d'*)			Response bias[b] (*c*)		
Measure	***M***	***SD***	***M***	***SD***	***M***	***SD***	***Skew***	***M***	***SD***	***Skew***
SDT-Math					1.89	0.81	-0.22	-0.60	0.31	0.27
Accuracy[a]	0.91	0.11	0.63	0.14						
RT (s)	1.02	0.16	1.25	0.23						
SDT-Punctuation					2.45	0.49	-0.29	-0.14	0.30	-0.23
Accuracy[a]	0.90	0.05	0.86	0.08						
RT (s)	1.07	0.18	1.14	0.18						

Note. *N* = 58.

[a]Proportion correct. [b]See text for description.

Response time — Participants were faster to judge conventional than non-conventional stimuli (1.05 vs. 1.19s), $F(1,57) = 102.16$, $p < .001$, $\eta_p^2 = 0.64$. However, the average response time for judging mathematical stimuli (*M* = 1.14s) was not significantly different from that for judging punctuation stimuli (*M* = 1.11s), $F(1, 57) = 2.68$, $p = .11$, $\eta_p^2 = 0.05$. There was a significant interaction between task and stimulus type, $F(1,57) = 24.29$, $p < .001$, $\eta_p^2 = 0.30$. Participants were significantly slower judging non-conventional mathematical stimuli compared to judging non-conventional punctuation stimuli (*M* = 1.25s, *M* = 1.14s respectively; $p < .001$) whereas their response times were not significantly different when judging conventional stimuli (*M* = 1.02s, *M* = 1.07s respectively; $p = .15$). Response times were not used for further analyses because of the high error rate on non-conventional items for the SDT-Math (see next section).

Accuracy — Participants were more accurate at judging the conventionality of punctuation than of mathematical symbol combinations (0.88 vs. 0.77), $F(1,57) = 57.73$, $p < .001$, $\eta_p^2 = 0.50$. There was a significant interaction between task and stimulus type, $F(1,57) = 119.59$, $p < .001$, $\eta_p^2 = 0.68$. Post-hoc analyses confirmed that participants were less accurate on non-conventional items for the SDT-Math than for the SDT-Punctuation (*M* = 0.63, *M* = 0.86 respectively; $p = .04$) whereas performance on conventional items was very similar for SDT-Math and SDT-Punctuation (*M* = 0.91, *M* = .90 respectively; $p = .96$).

Accuracy on the non-conventional items for the SDT-Math was quite low and therefore sensitivity scoring was used to correct for potential guessing (Diependaele et al., 2012). For consistency, the same scoring was used for SDT-Punctuation, although accuracy on the non-conventional items was high. For both measures, *d'* was positive, indicating that participants were able to discriminate between the signal (correctly identifying conventional items) and noise (incorrectly identifying non-conventional items as conventional) at above chance levels. Further, *d'* scores fell within typical performance levels (0.26 to 3.63) for sensitivity scoring and notably did not approach the maximum *d'* score of 4.65 (Keating, 2005). Interestingly, the measure of bias (*c*) was negative for both symbol-decision tasks, indicating that the participants were more likely to judge stimuli as conventional than as non-conventional (Stanislaw & Todorov, 1999).

Mathematical and Working Memory Task Performance

Performance on the mathematical outcome tasks and the domain-general cognitive control measures is shown in Table 2. Consistent with other research, accuracy on the basic digit tasks (i.e., digit comparison and order judgments) was very high (i.e., 94% and 99%, respectively) and therefore we used an inverse efficiency score to index performance (i.e., total RT/total correct). Although a few participants had quite high scores, arithmetic fluency performance was at the mean for this population (Sowinski, Dunbar, & LeFevre, 2014).

Table 2

Descriptive Statistics

Measure	Max Possible	*M*	*SD*	Min	Max	z_{Skew}
Digit Comparison[a]	–	0.68	0.08	0.52	0.88	1.65
Order judgments[b]	–	2.09	0.69	0.89	4.15	2.52*
Arithmetic fluency[c]	180.00	30.36	14.55	7.00	75.00	2.53*
Fraction/algebra procedures[c]	10.00	4.50	2.34	1.00	10.00	1.61
Word problem solving[c]	16.00	10.29	3.21	1.00	15.00	-2.93*
Number line[d]	–	6.75	3.90	3.13	18.13	4.98*
Backwards digit span[c]	18.00	10.29	2.38	4.00	14.00	-1.26
Spatial span[c]	18.00	10.53	2.46	4.00	15.00	-2.25*

Note. *N* = 58.

[a](mean response time)/(mean accuracy). [b](total response time)/(total accuracy). [c]Number correct. [d]Mean percent absolute error.

*$p < .05$.

Rational number arithmetic and algebra are challenging for many adults (Siegler & Lortie-Forgues, 2017). Accordingly, in the present study, the mean on the fraction arithmetic/algebra measure was 4.5 out of 10. Nevertheless, the task performance was normally distributed, and participants' scores covered a large range (i.e., 1 to 10). The distribution of the number line data was slightly skewed which is typical; many participants had low error scores, but a few people performed more poorly and thus had higher error scores. These participants may have struggled identifying the midpoint in the 0-7000 number line (Di Lonardo et al., 2020). In general, the tasks showed reasonable range and variability.

Correlations Amongst Tasks

Correlations amongst all of the measures are shown in Table 3. The STEM variable was coded 0 for students in Arts, Social Sciences, and Business majors and 1 for students in Science, Technology, Engineering, and Math (STEM) majors. As predicted, SDT-Math performance was significantly correlated with all other numerical measures whereas SDT-Punctuation performance was not significantly correlated with any of the numerical measures. Moreover, the SDT-Math was positively correlated with enrollment in STEM fields, indicating that

STEM students were better at discriminating between conventional and non-conventional mathematical orthography, whereas the SDT-Punctuation was negatively correlated with enrollment in STEM, indicating that students in Arts, Social Sciences, and Business were better at discriminating between conventional and non-conventional punctuation. These patterns support the view that the SDT-Math taps into participants' familiarity and experience with mathematical text whereas the SDT-Punctuation taps into participants' familiarity and experience with lexical text. The absence of a significant correlation between the SDT-Math and SDT-Punctuation was unexpected, given the similarity in the structure of these tasks, but supports the view that these measures index participants' familiarity with orthography specific to a given writing system.

Table 3

Zero-Order Correlations Among the Math Outcomes, Experimental Tasks and Control Variables

Measure	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1. SDT-Math[a]										
2. SDT-Punctuation[a]	-.11									
3. STEM[b]	.50**	-.30*								
4. Digit Comparison[c]	-.25†	-.11	-.05							
5. Order Judgments[d]	-.47**	-.09	-.21	.67**						
6. Arithmetic Fluency[e]	.50**	-.04	.25	-.29*	-.61**					
7. Fraction/algebra Procedures[e]	.62**	-.12	.52**	-.32*	-.50**	.60**				
8. Number Line[f]	-.41**	-.20	-.14	.37**	.34*	-.26*	-.31*			
9. Word-problem Solving[e]	.48**	.22	.17	-.42**	-.48**	.35**	.53**	-.47**		
10. Backwards Digit Span[e]	.36*	.30*	.19	-.18	-.32*	.28*	.31*	-.48**	.51**	
11. Spatial Span[e]	.28*	.16	.10	-.56**	-.44**	.01	.20	-.35**	.28*	.24

Note. *N* = 58.

[a]*d*′ sensitivity score. [b]Non-STEM major = 0, STEM major = 1. [c](mean response time)/(mean accuracy). [d](total response time)/(total accuracy). [e]total correct. [f]mean percent absolute error.

†*p* = .05. **p* < .05. ***p* < .01.

Both the SDT-Math and SDT-Punctuation were correlated with backwards digit span but only the SDT-Math was correlated with spatial span. These patterns suggest that both tasks involved verbal working memory processes, but that only the SDT-Math requires spatial processes (Morsanyi et al., 2017). The relation between mathematical orthography and spatial memory is consistent with other research linking math and spatial skills (see Mix et al., 2016 for a review). The symbol-decision task for punctuation (SDT-Punctuation) was excluded from subsequent regression analyses because it was not significantly correlated with the number tasks or the mathematical outcomes.

We replicated patterns of significant correlations among digit comparison, order judgments, and arithmetic fluency that are found in other studies (Lyons & Beilock, 2011; Sasanguie et al., 2017; Vogel et al., 2019; Xu et al., 2019). Furthermore, although participants' digit comparison and order judgment scores were both significantly correlated with arithmetic fluency, the latter correlation was much higher than the former. Thus, the present data show patterns of relations among the key number processing measures that are similar to those in previous research.

The strength of correlations amongst the working memory measures and outcome measures were comparable to those in other studies (Peng et al., 2016). Backwards digit span was correlated with all outcome measures whereas spatial span was correlated with performance on the number line task and word problem solving. Although not significant, the strength of the correlation between spatial span and fraction/algebra procedures (r = 0.20) was similar to that in a meta-analysis of previous findings between spatial span and similar mathematical outcomes (see Peng et al., 2016). Also, the difference in correlational patterns for working memory measures and arithmetic (i.e., backwards digit span was correlated with arithmetic whereas spatial span was not) was consistent with findings that phonological processes such as fact retrieval are more strongly related to adults' calculation skills whereas visual-spatial processes are more strongly related to children's arithmetic skills (McKenzie et al., 2003; Raghubar et al., 2010; Rasmussen & Bisanz, 2005). Together the strength and patterns of the correlations between working memory measures and the mathematical outcome variables support using these measures as control variables.

As anticipated, the correlation between the SDT-Math and order judgments was higher than that between SDT-Math and digit comparison, z = -2.19, p = .028 (Lyons & Beilock, 2011; Sasanguie et al., 2017). This pattern is consistent with the view that mathematical orthography involves hierarchically more advanced symbol relations (Xu et al., 2019). In summary, the correlations between the SDT-Math and the other mathematical tasks show that this measure captures math-relevant individual differences. In the next section, we explored the predictors of the SDT-Math task before turning to multiple regressions of the mathematical outcomes.

Regression Analyses of SDT-Mathematics

We begin with an exploratory regression to better understand the skills most closely associated with the mathematical orthography task. We simultaneously regressed the SDT-Math on field of study, working memory, and basic number knowledge. As shown in Table 4, enrollment in STEM fields was a significant predictor, consistent with the assumption that the SDT-Math is an index of familiarity with mathematical symbols. Ordinal knowledge was also a significant predictor, presumably reflecting the close ties among fundamental measures of associative knowledge of mathematical symbols (Xu et al., 2019).

Table 4

Regression Model for the Symbol Decision Task-Math Including the Inclusion Bayes Factor

Measure	Standardized Coefficients	Correlations		Unique R^2 (%)	BF_{inc}
		Partial	Part		
STEM field	.40**	.46**	.39**	15.2**	98.1
Backward span	.20	.24	.19	3.6	1.6
Spatial span	.09	.10	.07	0.5	0.4
Digit comparison	.09	.08	.06	0.4	0.4
Order judgments	-.33*	-.29*	-.23*	5.5*	5.6
Total R^2 (%)	42.3**				
F model	7.47**				
F (df)	(5, 51)				

Note. Correlations and unique R^2 values are for the final model. BF_{inc} is the inclusion Bayes Factor.
*p < .05. **p < .01.

The Bayesian linear regression indicated that there was strong evidence (BF_M = 11.9) for the model that included three predictors of the SDT-Math, that is, STEM status, ordinal skills, and backwards digit span. Notably, there was anecdotal evidence for including the backwards digit span in the model whereas there was strong evidence for including ordinal skills and decisive evidence for the inclusion of STEM majors (see BF_{inc} values in Table 4). Thus, as we predicted, the SDT-Math captures participants' familiarity with the symbols required for activities such as solving algebra problems, knowledge that is more familiar to students majoring in STEM disciplines such as math and science.

Multiple Regression Analyses of Outcomes

In this section, we test the regression models proposed in the introduction to evaluate the hypothesis that the SDT-Math captures the variance in the advanced outcomes beyond that associated with digit-symbol associations and working memory. Four mathematical outcomes, that is, arithmetic fluency, fraction/algebra procedures, word problem solving, and number line estimation, were regressed on the predictors. To reduce variability in the outcome measures, both working memory measures were included in the regressions. Analyses are shown in Table 5, Table 6, Table 7, and Table 8. In each table, the standardized coefficients of the hierarchical regressions are shown (i.e., Models 1 through 4) as each additional predictor was added to the analysis. The second-last column of each table is the unique variance in the outcome that is associated with each predictor (i.e., in the final model). Hierarchical regressions were used to demonstrate patterns of shared versus unique variance because all of the numerical predictors (i.e., digit comparison, order judgments, and SDT-Math) had significant simple correlations with the outcome variables. Notably, based on post-hoc power analyses each regression has a statistical power > 0.90 (Soper, 2019).

Table 5

Regression Models for Arithmetic Fluency

	Standardized coefficients and estimates				Correlations			
Predictor	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Partial**	**Part**	**Unique R^2 (%)**	**BF_{inc}**
Backward span	.27*	.26*	.11	.06	.07	.05	0.2	0.3
Spatial span	-.05	-.23	-.28*	-.32*	-.34*	-.26*	6.5*	7.6
Digit comparison		-.32*	.14	.09	.09	.06	0.4	0.4
Order judgments			-.78**	-.66**	-.54**	-.45**	19.8**	1839.6
SDT-Math				.27*	.31*	.23*	5.1*	3.7
Total R^2 (%)	7.1	14.2*	45.8**	50.9*				
F model	2.07	2.93*	10.98**	10.58**				
F (*df*)	(2, 54)	(3, 53)	(4, 52)	(5, 51)				

Note. Part and partial correlations and unique R^2 values are for the final model. BF_{inc} is the inclusion Bayes Factor.
**p* < .05. ***p* < .01.

Arithmetic Fluency

To test the relations between mathematical orthography and arithmetic, we regressed arithmetic fluency on digit skills (i.e., digit comparison and order judgment) and the *d*' score for the symbol-decision task for math (SDT-Math). As shown in Table 5, the model explained 51% of the variance in arithmetic fluency. Notably, the predictive power of spatial span increased as other predictors were added to the model, thus spatial span was acting as a suppressor variable (Ludlow & Klein, 2014). However, excluding spatial span from the regression did not change the overall pattern of results. As in previous research, order judgments accounted for substantial unique variance in arithmetic (e.g., Lyons & Beilock, 2011; Xu et al., 2019) but as we predicted, the SDT-Math also accounts for unique variance in the final model. This finding supports the hypothesis that individual differences in arithmetic fluency are related to participants' mathematical orthography.

As in several previous studies, digit comparison was not significantly related to arithmetic fluency once order judgments were included (e.g., see Model 3; Lyons & Beilock, 2011; Sasanguie et al., 2017). Further, Bayesian regression indicated that there was strong evidence (BF_m = 22.8) that spatial span, order judgments, and the SDT-Math provided the best fitting model for arithmetic fluency. Moreover, the Bayesian inclusion factors indicate that there was substantial evidence for including spatial span and the SDT-Math and decisive evidence for including order judgement in the regression. In summary, the regression analyses replicate previous findings for the relations between arithmetic and digit associations and support the main hypothesis that the mathematical symbol-decision task explained individual differences in arithmetic fluency beyond those differences accounted for by symbolic digit skills.

Fraction/Algebra Procedures

We hypothesized that the SDT-Math and arithmetic fluency would account for unique variance in fraction/algebra procedures. Further, any relations between basic digit skills and fraction/algebra procedures would be accounted for by shared variance with arithmetic and SDT-Math. As shown in Table 6, our hypotheses were supported. Both arithmetic fluency and mathematical orthography uniquely predicted performance on the fraction/algebra measure. Moreover, the Bayesian regression provided strong evidence (BF_M = 28.1) that arithmetic fluency and mathematical orthography provided the best fitting model for the data with very strong support for including the SDT-Math in the regression.

Table 6

Regression Model for Fraction/Algebra Procedures

	Standardized coefficients and estimates				Correlations		Unique R^2	
Predictor	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Partial**	**Part**	**(%)**	BF_{inc}
Backward digit span	.26†	.16	.11	.04	.05	.04	0.1	0.3
Spatial span	.14	-.02	.11	.02	.03	.02	0.0	0.3
Digit comparison		.06	-.01	-.06	-.06	-.04	0.2	0.3
Order judgments		-.48**	-.11	-.04	-.03	-.02	0.1	0.4
Arithmetic Fluency			.48**	.33*	.31*	.23*	5.5*	8.2
SDT-Math				.40**	.41**	.32**	10.3**	49.4
Total R^2 (%)	11.8	24.5**	37.8**	48.1**				
F model	3.12	4.47**	6.20**	7.73**				
F (*df*)	(2, 54)	(4, 52)	(5, 51)	(6, 50)				

Note. Partial and part correlations and unique R^2 values are for the final model. BF_{inc} is the inclusion Bayes Factor.
†p = .05. *p < .05. **p < .01.

Word Problem-Solving

As shown in Table 7, mathematical orthography accounted for unique variance in word problem solving, as did verbal working memory (i.e., backwards digit span). Because the problems were read to the participants, the verbal working memory load of this task was presumably higher than that of the other mathematical measures. Although neither arithmetic fluency nor cardinal and ordinal skills predicted unique variance, these predictors had significant zero-order correlations with word problem solving (see Table 3) and together accounted for considerable shared variance (i.e., 6%). Also, the Bayesian regression provided strong evidence (BF_M = 20.5) that backward digit span, cardinal skills, and the SDT-Math provided the best fitting model for the data. Further, there was strong evidence for including backward digit and substantial evidence for both cardinal skills and

SDT-Math. As predicted, the SDT-Math was a significant unique predictor of individual differences in word problem solving.

Table 7

Multiple Regression Models for Word Problem Solving

	Standardized coefficients and estimates				Correlations			
Predictor	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Partial**	**Part**	**Unique R^2 (%)**	**BF_{inc}**
Backward span	.47**	.41**	.40**	.35**	.40**	.32**	10.4**	21.0
Spatial span	.17	-.05	-.01	-.08	-.08	-.06	0.4	0.3
Digit comparison		-.25	-.26	-.30	-.27	-.20	4.1	3.1
Order judgments		-.21	-.12	-.07	-.06	-.04	0.2	0.6
Arithmetic Fluency			.12	.02	.02	.01	0.0	0.4
SDT-Math				.28*	.29*	.22*	5.0*	3.3
Total R^2 (%)	28.5	40.1	40.8	45.8				
F model	10.74**	8.70**	7.04**	7.06**				
F (*df*)	(2, 54)	(4, 52)	(5, 51)	(6, 50)				

Note. Part and partial correlations and uunique R^2 values are for the final model. BF_{inc} is the inclusion Bayes Factor.
*$p < .05$. **$p < .01$.

Number Line Estimation

Number line performance calls upon spatial skills and participants' understanding of the number system but does not require more advanced symbolic knowledge (LeFevre et al., 2013; Xu, 2019). However, the 0-7000 number line task used in the present study is more challenging for adults than typical number line tasks which have canonical endpoints (e.g., 1000 or 10000; Di Lonardo et al., 2020). Thus, we anticipated working memory may be implicated as participants might be referencing a calculated midpoint (7000 ÷ 2) versus a retrieved midpoint (1000 ÷ 2) to estimate the position of the target number on the number line (Di Lonardo et al., 2020). As shown in Table 8, both measures of working memory accounted for variance in number line performance in Model 1.

Table 8

Regression Model for Number Line Accuracy (Percentage Absolute Error)

	Standardized Coefficients and Model Estimates				Correlations			
Predictor	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Partial**	**Part**	**Unique R^2 (%)**	**BF_{inc}**
Backward span	-.43**	-.42**	-.40**	-.36**	-.40**	-.33**	10.8**	20.1
Spatial span	-.25*	-.09	-.15	-.10	-.10	-.07	0.5	0.5
Digit comparison		.29	.32	.35*	.29*	.24*	5.5*	3.4
Order judgments		-.01	-.17	-.21	-.16	-.12	1.5	0.4
Arithmetic fluency			-.22	-.13	-.12	-.09	0.8	0.4
SDT-Math				-.24	-.25	-.19	3.8	1.8
Model R^2 (%)	29.8**	35.3**	37.8**	41.6**				
F	11.5**	7.1**	6.2**	5.9**				
F (*df*)	(2, 54)	(4, 52)	(5, 51)	(6, 50)				

Note. Correlations and unique R^2 values are for the final model. BF_{inc} is the inclusion Bayes Factor.
*$p < .05$. **$p < .01$.

In the final model, digit comparison was the only unique predictor of number line performance other than backwards digit span. Interestingly, the Bayesian regression provided strong evidence (BF_M = 14.8) that, in addition to backwards digit span and digit comparison, the best fitting model included mathematical orthography. However, because the evidence for including SDT-Math in the regression model was anecdotal (BF_{inc} < 3) and the SDT was not a unique predictor (p = .08), we concluded, as expected, there is limited evidence that more advanced symbol knowledge (i.e., the SDT versus cardinal skills) predicts individual differences in number line estimation.

Discussion

Knowledge of written mathematical symbols is essential to mathematical learning and understanding. For example, the speed and accuracy of adults' judgments about cardinality as measured by digit comparison (Schneider et al., 2017), and the speed and the accuracy of adult's judgement about ordinality as measured by symbolic order judgments (Lyons et al., 2016), are well-established predictors of individual differences in arithmetic fluency. The hierarchy of symbol integration model (HSI) proposed by Xu et al. (2019) describes how associations between number symbols (cardinal and ordinal) predict mathematical outcomes. We extended the HSI model to include patterns of associations between more advanced symbols, specifically, mathematical orthography and mathematical outcomes. We tested these predictions using Bayesian and hierarchical linear regressions. We found that mathematical orthography, operationalized as the SDT-Math: (a) was correlated with all mathematical tasks, (b) accounted for some of the variance in the well-established relation between order judgments and arithmetic fluency, (c) predicted unique variance in all mathematical tasks except for number line accuracy, and (d) accounted for some of the variance in the relations between ordinal knowledge, arithmetic, and both fraction/algebra arithmetic and word problem solving. These findings indicate that the SDT-Math captures a previously unassessed source of individual differences in mathematical skills.

The role of symbolic associations among digits (e.g., digit comparison, order judgments, and arithmetic fluency) in understanding individual differences in mathematical knowledge is a central aspect of current research in mathematical cognition (Lyons et al., 2016; Lyons & Ansari, 2015). The present findings suggest that learners' familiarity with more advanced symbols are also relevant for models of mathematical knowledge. Order judgment tasks are robust predictors of arithmetic associations for adults (Lyons & Beilock, 2011; Sasanguie et al., 2017; Vos et al., 2017) and for children from grade 2 on (Lyons et al., 2014; Sasanguie & Vos, 2018; Xu & LeFevre, in press). However, the stored knowledge accessed in the order judgment task is limited to digit-digit associations. In the current research, order judgments of digits were not unique predictors of either fraction/algebra procedures or word problem solving whereas mathematical orthography (i.e., judgments of algebraic text) did capture unique variance in these outcomes. Thus, it is important to go beyond associations among digits to understand the role of symbolic knowledge in mathematical cognition.

Limitations and Further Directions

We recognized three limitations in the current study: sample size, the choice of domain-general controls, and the breadth of the SDT stimuli. First, although similar to that used in other studies of symbol knowledge (i.e., Lyons & Beilock, 2011; Sasanguie et al., 2017; Vogel et al., 2019), the sample size in the present research was modest. Thus, Bayesian analyses were used to complement and provide further support for the research findings. Additional studies, with larger sample sizes, will be critical to ascertaining whether, or under what conditions, the findings can be consistently replicated. One strength of this research is that multiple outcome measures were used, increasing our confidence that the relations between the SDT-Math and mathematical performance, more generally, is stable.

Second, the use of the symbol decision task for punctuation (SDT-Punctuation) as a control for orthographic skill, more generally, may have limited our conclusions. However, we did find the intriguing pattern that SDT-Punctuation was better for students in non-STEM disciplines whereas SDT-Math was better for students in STEM disciplines. This pattern suggests that each SDT requires knowledge that is specific to an orthographic domain and thus related to domain-relevant experience. However, the complete lack of correlation between the two orthographic judgment tasks was unexpected, given they have very similar features beyond the domain-specific content. Further research with the SDT-Punctuation and other similar measures in relation to reading comprehension or related outcomes will be helpful in establishing the best control for task-specific aspects of these orthography tasks.

Finally, the lack of breadth of mathematical symbols in the SDT-Math is a potential limitation. By design, the SDT-M task as a measure of mathematical orthography was based on an analysis of pre-algebra and algebra curricula (Headley, 2016). We assume that during high school, all of the participants in the present research were exposed to symbols and combinations of symbols that are similar to those in the conventional stimuli. Their relatively poor performance on the non-conventional combinations of symbols suggests that many of these adults never mastered or cannot recall the conventions for combining these mathematical symbols. The results may be different for groups of adults who are highly skilled and/or have much more frequent and recent exposure to these and other symbols. Consistent with this view, we found that students who are majoring in STEM disciplines (e.g., Science, Technology, Engineering, or Mathematics) had higher scores on the SDT-Math than students majoring in Arts, Social Sciences, or Business. These symbols are commonly encountered in university level mathematics and science courses. In summary, this study supports the assumption that the SDT-Math indexes familiarity with the orthographic conventions of the mathematical writing system, but conclusions may be limited to a subset of the possible stimuli.

Importantly, this research was not intended to address the claim that students' knowledge of mathematical symbols and writing conventions is a necessary condition for advanced mathematics achievement. Students' familiarity with mathematical orthography is likely to emerge in reciprocal transaction (Baltes et al., 2006) between mathematics instruction and their engagement with mathematical activities. The SDT-Math was intentionally designed to avoid any requirements for participants to compare magnitudes, evaluate a numeric expression, or determine the validity of an equation or relationship. Thus, it does not conflate adults' mathematical orthography with their ability to do mathematics. Future studies might include an exploration of whether mathematical orthography is correlated with the ability to evaluate or compare numeric expressions. For example, it would be useful to know if students who have difficulty identifying inequalities such as $8 < 5$ as a false statement also have difficulty recognizing text such as $8 \vee 5$ as non-conventional. Further research with students at various stages of learning will be necessary to firmly establish the place of mathematical orthography in the broader domain of mathematical knowledge.

Conclusions

The present research illustrates how using innovative interdisciplinary theoretical perspectives – in this case, applying a reading research perspective to investigate the unique literacy demands of mathematics cognition – can carve out new pathways for understanding the complex combination of skills that contribute to mathematics achievement. Our findings provide novel insights into individual differences in the relations between mathematical symbol knowledge and mathematical skills. These findings are consistent with the view that mathematical symbol knowledge can be modelled as hierarchically organized networks of associations (Hiebert, 1988; Xu et al., 2019). We extended the view of symbol knowledge from associations amongst digits (i.e., cardinal and ordinal) to include associations amongst a broader set of mathematical symbols. This research establishes the symbol-decision task as a useful index of relevant individual differences in adults' mathematical orthography that goes beyond digits.

The current results, if borne out in future studies of mathematical orthography, have implications for education. Extant research shows that knowledge of non-numeric symbols (e.g., equal signs, operation signs, fraction bars, decimal points) is relevant to elementary mathematical development. However, the cognitive underpinnings of advanced mathematical orthography and their relations to the development of secondary mathematical skills is not well-understood. Although mathematical orthography is only a small portion of the comprehensive math-related skills that educators hope their students will develop, some educators argue that teaching symbols and symbol conventions is a critical part of mathematics instruction (Bardini & Pierce, 2015; Chirume, 2012; Quinnell & Carter, 2012; Rubenstein & Thompson, 2001). In combination, their reasoning and these results suggest it is critical to advance research aimed at answering questions about how and the degree to which explicit teaching of mathematical orthography (i.e., symbols and symbol conventions) supports the emergence of orthographic knowledge of symbolic mathematics and advanced mathematical development. By introducing an initial theoretical definition of mathematical orthography and a measurement tool to operationalize it, the present research makes a significant first step towards understanding it as a unique source of individual differences in mathematical skill.

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Competing Interests

The authors have declared that no competing interests exist.

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The authors have no support to report.

Data Availability

For this study, a dataset is freely available (Douglas, Headley, Hadden, & LeFevre, 2020).

Supplementary Materials

The dataset used for this paper is freely available on the OSF repository (Douglas, Headley, Hadden, & LeFevre, 2020).

Index of Supplementary Materials

Douglas, H., Headley, M. G., Hadden, S., & LeFevre, J.-A. (2020). *Supplementary materials to "Knowledge of mathematical symbols goes beyond numbers"* [Research data]. OSF. https://osf.io/58cma

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Appendices

Appendix A

Table A.1

Full Test Battery in Order of Test Administration

Measure	Focus
Math background and interest questionnaire	Background survey
Arithmetic fluency	Mathematical outcome
Peabody Picture Vocabulary Test[a]	Language skills
Phonological awareness[a]	Language skills
Symbol-decision task – Math	Mathematical Orthography
Symbol-decision task – Punctuation	Lexical Orthography
Backwards digit span	Attention and working memory
Black and white Stroop task[a]	Attention and working memory
Spatial span	Attention and working memory
Digit comparison	Digit knowledge
Number ordering	Digit knowledge
Order judgments	Digit knowledge
Number-line estimation	Mathematical outcome
KeyMath applied problem solving	Mathematical outcome
Mental rotation[a]	Spatial skills
Spatial orientation[a]	Spatial skills

[a]Not used in current analyses.

Appendix B

Item #s	Conventional	Unconventional	Characteristics	
			Ink Marks	2
1, 2	x^2	${}^2x$	**Orthographic change**	Location(exp/sub)
			Curriculum	Function
			Ink Marks	3
3, 4	$\lvert x \rvert$	$x\lvert\,\rvert$	**Orthographic change**	Location(bracket)
			Curriculum	Function
			Ink Marks	3
5, 6	${}^{x}\!/_{y}$	${}_{y}\!\diagdown^{x}$	**Orthographic change**	Orientation(operator)
			Curriculum	Operation
			Ink Marks	3
7, 8	$f(x)$	$(f)x$	**Orthographic change**	Location(bracket)
			Curriculum	Function
			Ink Marks	2
9, 10	$\sqrt{x}$	$\overline{)x}$	**Orthographic change**	Shape(function)
			Curriculum	Function
			Ink Marks	3
11, 12	$l \times w$	$l \otimes w$	**Orthographic change**	Shape(operator)
			Curriculum	Operation(simple)
			Ink Marks	5
13, 14	$a > b > c$	$a < b > c$	**Orthographic change**	Orientation(relationship)
			Curriculum	Statements
			Ink Marks	4
15, 16	$x \leq y$	$x \ll y$	**Orthographic change**	Shape(relationship)
			Curriculum	Statements
			Ink Marks	6
17, 18	$(x-y)^2$	$(x-y)_2$	**Orthographic change**	Location(exp/sub)
			Curriculum	Operation(complex)
			Ink Marks	4
19, 20	x^{m+n}	xm^{+n}	**Orthographic change**	size(variable)
			Curriculum	Operation(complex)

Table B.1. Experimental Stimuli for the Symbol-Decision Task for Math (SDT-Math)

Items #	Conventional	Unconventional	Characteristics	
21, 22	$x - x_1$	$x - {}_1x$	**Ink Marks**	4
			Orthographic change	Location(exp/sub)
			Curriculum	Operation(simple)
23, 24	$a \cdot b$	$a.b$	**Ink Marks**	3
			Orthographic change	Location(operator)
			Curriculum	Operation
25, 26	πr^2	${}_2\pi r$	**Ink Marks**	3
			Orthographic change	Location(exp/sub)
			Curriculum	Operation(complex)
27, 28	$2x$	$2\times$	**Ink Marks**	2
			Orthographic change	Shape(operator)
			Curriculum	Operation(simple)
29, 30	$d = rt$	$d \parallel rt$	**Ink Marks**	5
			Orthographic change	Orientation(relationship)
			Curriculum	Statements
31, 32	y^3	${}^3y$	**Ink Marks**	2
			Orthographic change	Location(exp/sub)
			Curriculum	Function
33, 34	$\lvert y \rvert$	$\lvert\lvert y$	**Ink Marks**	3
			Orthographic change	Location(bracket)
			Curriculum	Function
35, 36	${}^{a}/_{b}$	${}_{b}\backslash^{a}$	**Ink Marks**	3
			Orthographic change	Orientation(operator)
			Curriculum	Operation(simple)
37, 38	$g(x)$	$(g)x$	**Ink Marks**	3
			Orthographic change	Location(bracket)
			Curriculum	Function
39, 40	$\sqrt{y}$	$\overline{)y}$	**Ink Marks**	2
			Orthographic change	Shape(function)
			Curriculum	Function

Table B.1 [continued]. Experimental Stimuli for the Symbol-Decision Task for Math (SDT-Math)

Items #	Conventional	Unconventional	Characteristics	
41, 42	$p \div q$	$p \because q$	**Ink Marks**	5
			Orthographic change	Shape(operator)
			Curriculum	Operation(simple)
43, 44	$x > y > z$	$x < y > z$	**Ink Marks**	5
			Orthographic change	Orientation(relationship)
			Curriculum	Statements
45, 46	$x \geq y$	$x \gg y$	**Ink Marks**	4
			Orthographic change	Shape(relationship)
			Curriculum	Statements
47, 48	$(x-y)^3$	${}^3(x-y)$	**Ink Marks**	6
			Orthographic change	Location(exp/sub)
			Curriculum	Operation(complex)
49, 50	y^{a+b}	ya^{+b}	**Ink Marks**	4
			Orthographic change	size(variable)
			Curriculum	Operation(complex)
51, 52	$y_2 - y_1$	$y_2 - {}_1y$	**Ink Marks**	5
			Orthographic change	Location(exp/sub)
			Curriculum	Operation(simple)
53, 54	$x \cdot y$	$x.y$	**Ink Marks**	3
			Orthographic change	Location(operator)
			Curriculum	Operation
55, 56	π^2	${}^2\pi$	**Ink Marks**	3
			Orthographic change	Location(exp/sub)
			Curriculum	Operation(complex)
57, 58	2^x	$2^\times$	**Ink Marks**	2
			Orthographic change	Shape(operator)
			Curriculum	Operation(simple)
59, 60	$y = \frac{c}{x}$	$y = \overset{c}{\overline{)x}}$	**Ink Marks**	6
			Orthographic change	Shape(operator)
			Curriculum	Statements

Table B.1 [continued]. Experimental Stimuli for the Symbol-Decision Task for Math (SDT-Math)

Appendix C

Table C.1

Experimental Stimuli for the Symbol-Decision Task for Punctuation (SDT-Punctuation)

Items 1 to 30		Items 31 to 60	
Conventional	**Non-conventional**	**Conventional**	**Non-conventional**
1. haven’t	2. haven’’t	31. Caution: slippery	32. Caution slippery:
3. semi-annual	4. semi:annual	33. Superb!	34. !Superb
5. e.g.	6. e.g -	35. his & hers	36. his hers&
7. Where?	8. Where&	37. “Goodbye”	38. “Good”bye
9. furthermore,	10. furthermore!	39. ; hence	40. hence ;
11. Note:	12. Note’	41. o’clock	42. oʻclock
13. Wow!	14. Wow,	43. time-out	44. time_out
15. this & that	16. this ? that	45. Done.	46. Done·
17. “Hello”	18. ;Hello;	47. What?	48. What¿
19. However;	20. However.	49. then,	50. then’
21. don’t	22. do’nt	51. Warning:	52. Warning··
23. part-time	24. partt-ime	53. Yahoo!	54. Yahoo¡
25. i.e.	26. .ie.	55. rock & roll	56. rock ⅋ roll
27. How?	28. ?How	57. “Okay”	58. „Okay„
29. therefore,	30. there,fore	59. Meanwhile;	60. Meanwhile.,

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