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Flexibility and Adaptivity in Arithmetic Strategy Use: What Children Know and What They Show

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Abstract

Central elements of adaptive expertise in arithmetic problem solving are flexibility, using multiple strategies, and adaptivity, selecting the optimal strategy. Research shows that the strategies children actually use do not fully reflect the strategies they know: there is hidden potential. In the current study a sample of 147 third graders from the Netherlands completed a comprehensive assessment of adaptive expertise in the domain of multidigit subtraction, designed to measure, first, the strategies students know and use to solve subtraction problems (potential and practical flexibility). Second, it measured to what extent students know which strategy is optimal and to what extent they use the optimal strategy (potential and practical adaptivity). Findings for flexibility showed that most students consistently used the same strategy across all problems: practical flexibility was low. When prompted, students knew more strategies than they used spontaneously, suggesting hidden potential in flexibility. Findings for adaptivity showed that students hardly ever spontaneously used the task-specific strategy that is efficient for specific problems since it has the fewest and easiest steps. However, almost half of the students could select this strategy from a set of given strategies at least once. Furthermore, an innovative, personalized version of the choice/no-choice method showed that the task-specific strategy was usually not the optimal strategy (fastest strategy leading to a correct answer) for individual students. Finally, students used the strategy with which they performed best more often than the other strategies, but there is hidden potential for the adaptive use of task-specific strategies.

Keywords

mathematics, subtraction, strategies, flexibility, adaptivity, adaptive expertise, primary school

There are various ways to solve mathematics problems such as 84 − 67 = ?, for instance adding-on from 67, or sequentially subtracting 60 and 7 (Blöte et al., 2001; Threlfall, 2009; Torbeyns et al., 2009). Throughout the world mathematics educators stress the importance of children's adaptive expertise, with the ability to solve such mathematics problems flexibly, with a variety of meaningful strategies, and adaptively, by selecting the optimal strategy, as central elements (Baroody, 2003; Heinze et al., 2018; McMullen et al., 2016; Torbeyns et al., 2009a). Adaptive expertise is an indicator of deeper mathematical understanding and a key aspect in the development of later mathematical competence and success (Star et al., 2015; Xu et al., 2017). However, research shows that children's level of adaptive expertise is rather disappointing: students tend to quite consistently use the same strategy across problems and efficient shortcut strategies are rarely used (De Smedt et al., 2010; Hickendorff, 2018; Selter, 2001; Torbeyns et al., 2017; Xu et al., 2017).

Importantly, flexibility research shows that what children do on a task, their actual strategy use, does not fully reflect what they know (Blöte et al., 2001; Torbeyns et al., 2009b; Xu et al., 2017). Therefore, it is relevant to distinguish between knowing and using multiple strategies (Newton et al., 2020; Xu et al., 2017). The current paper therefore aims to chart not only students' practical level of flexibility and adaptivity, but also their (hidden) potential, in the domain of multi-digit subtraction.

Multi-Digit Subtraction

There is near consensus on the strategies that can be used to solve multi-digit subtraction problems, see Hickendorff et al. (2019) for a recent overview. The current study focuses on mental or number-based strategies only, excluding digit-based procedures such as the written algorithm. There are four main number-based strategies for multi-digit subtraction (Table 1).

Table 1

The Four Main Subtraction Strategies

Strategy	Steps in solving 84 − 67 =
Jump	84 − 60 = 24; 24 − 7 = 17
Split	80 − 60 = 20; 4 − 7 = 3 short; 20 − 3 = 17
Compensation	84 − 70 = 14; 14 + 3 = 17
Indirect Addition	67 + 3 = 70; 70 + 14 = 84; 3 + 14 = 17

Jump and split strategies are universal strategies that can be used on all types of problems (Heinze et al., 2018). In the jump, or sequential, strategy, subtraction is seen as a backward movement of numbers on the (mental) number line (Torbeyns et al., 2017). By contrast, in the split, or decomposition, strategy, the decimal structure of numbers is used to partition both operands. This strategy is usually discouraged in the subtraction learning trajectory since it leads to problems when it is necessary to cross the tens, leading to a smaller-from-larger error where students switch the numbers in 4 − 7 to 7 − 4 in the example (Vermeulen et al., 2020).

Compensation and indirect addition are task-specific strategies (also called 'varying' strategies), since they are efficient on specific problems (Heinze et al., 2018). Compensation is particularly suited for problems in which the subtrahend is close to a round number, for instance because the units are 8 or 9 (Blöte et al., 2001; Heinze et al., 2018; Torbeyns et al., 2017). Indirect addition, also called subtraction-by-addition, is particularly suited for problems with a small difference between minuend and subtrahend (Hickendorff, 2020; Torbeyns et al., 2011, 2018).

Adaptive Expertise

Adaptive expertise concerns the ability to apply meaningfully learned procedures flexibly and creatively. It contrasts with routine expertise: simply being able to complete school mathematics exercises quickly and accurately without understanding (Verschaffel et al., 2009). In the current study we focus on flexibility and adaptivity as central elements of adaptive expertise, where flexibility refers to using multiple strategies and adaptivity to selecting the most appropriate, or optimal, strategy (Verschaffel et al., 2009). The definitions of adaptivity researchers use are different and sometimes even partly contradictory (Brezovszky et al., 2019; Verschaffel et al., 2009). Some researchers define the optimal strategy solely from the perspective of the task, as the strategy with the fewest and easiest steps (Blöte et al., 2001; Heinze et al., 2009; Hickendorff, 2018). Others define the optimal strategy as the strategy that works best on a given task for an individual child (Fagginger Auer et al., 2016; Siegler & Lemaire, 1997; Torbeyns et al., 2004). Importantly, these two definitions may designate different strategies as optimal, since not all children have the understanding required for the 'task-appropriate' strategy (Hickendorff, 2018; Torbeyns et al., 2009b), which complicates the integration of findings. In the current framework we therefore explicitly distinguish between these two types of adaptivity, yielding three components of adaptive expertise: flexibility, task-based adaptivity, and individual-based adaptivity.

Regarding flexibility and task-based adaptivity, research usually shows rather disappointing results: students tend to quite consistently use the same strategy across problems and efficient task-specific strategies, also called shortcut strategies, are rarely used (De Smedt et al., 2010; Hickendorff, 2018; Selter, 2001; Torbeyns et al., 2017; Xu et al., 2017). However, when asked to solve the same problem more than once, using alternative strategies, students use more different strategies than in only one attempt (Blöte et al., 2001; Newton et al., 2020; Torbeyns et al., 2009b; Xu et al., 2017). Furthermore, secondary students can select from a list of given correct strategies which strategy is 'innovative' to solve an equation (Newton et al., 2020; Xu et al., 2017). There is thus reason to expect that students know more than they show spontaneously: that there is hidden potential in both flexibility and task-based adaptivity. This potential has however not been investigated simultaneously in primary mathematics. Furthermore, whether students know which strategy is optimal for themselves (potential individual-based adaptivity) has not been investigated before.

Therefore, we propose a comprehensive conceptual model of adaptive expertise with six dimensions. *Practical flexibility* refers to using multiple strategies and *potential flexibility* to knowing multiple strategies. *Practical task-based adaptivity* refers to using the task-appropriate strategy and *potential task-based adaptivity* to knowing which strategy is most appropriate for a task. Finally, *practical individual-based adaptivity* refers to students using the strategy that is optimal for themselves on a given problem (fastest strategy leading to a correct answer), and *potential individual-based adaptivity* to students knowing which strategy is optimal for themselves on a given problem.

Individual Strategy Performance

To investigate which strategy is optimal for an individual, the choice/no-choice methodology has been developed (Siegler & Lemaire, 1997) and implemented in studies of mathematical strategies use (for an overview, see Luwel et al., 2009). Crucial to this methodology are the no-choice conditions, where participants must use pre-defined strategies to solve all problems. When participants are free to choose their strategies, strategy performance data are biased by selection effects. For instance, when strategies are used on simpler problems or by more skilled individuals the strategies seem more accurate than they are. When participants are obliged to use the same strategy on all problems in the no-choice conditions, these biases cannot occur. By comparing the performance (accuracy and speed) of different strategies from the no-choice conditions, it is possible to know which strategy is optimal for an individual on a given problem (i.e., which strategy leads to a correct answer fastest).

A major drawback, however, is that it has to be decided *a priori* which strategies are focal, so that the choice condition entails a forced choice rather than a free choice between strategies (Luwel et al., 2009; Threlfall, 2009). Consequently, we might underestimate children's strategy adaptivity: children might be able to efficiently and adaptively use other strategies that were not included in the design. In the current study we therefore personalized the no-choice conditions, as suggested by Hickendorff (2020). Students had to use the task-specific strategy in one no-choice condition and the strategy they used in the free choice condition in the other. This enables investigating the extent to which the task-specific strategy is also optimal for individual students, without restricting the range of possible strategies, and as such providing a more complete and ecologically valid picture of adaptivity.

Student Factors Related to Adaptive Expertise

There are several student factors that have been found to relate to aspects of adaptive expertise in arithmetic. One is students' mathematics ability. It is a common assumption that the acquisition of strategy flexibility is more difficult for lower achieving students (Geary, 2003; National Mathematics Advisory Panel, 2008). Although several studies indeed reported that students with higher mathematics achievement levels show higher levels of flexibility and task-based adaptivity than lower achievers (Hickendorff et al., 2010, 2018; Newton et al., 2020; Torbeyns et al., 2006, 2017), other studies showed no relation with mathematics level (Torbeyns et al., 2018). A second relevant individual difference involves a more specific component of mathematical knowledge: adaptive number knowledge. It refers to the well-connected knowledge of numerical characteristics and arithmetic relations, and has been found to be related to arithmetic fluency, knowledge of arithmetic concepts, and pre-algebra skills (Brezovszky et al., 2019; McMullen et al., 2016, 2017). Such an understanding of numbers and relations is probably particularly important for task-based adaptivity,

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where students must notice when and which strategies are applicable. However, the relation between adaptive number knowledge and strategy flexibility and adaptivity has not been empirically investigated so far.

Besides mathematical knowledge, domain-general cognitive resources are likely to be related to students' adaptive expertise. Recent meta-analyses showed that working memory, the ability to simultaneously store and process information (Baddeley, 1992), is related to mathematics performance (Friso-van den Bos et al., 2013; Peng et al., 2016). Working memory is likely to play a role in strategy flexibility and adaptivity, because different arithmetic strategies put different demands on working memory (DeStefano & LeFevre, 2004; Imbo & Vandierendonck, 2007; Threlfall, 2009). Finally, gender differences in flexibility and adaptivity have been found. Girls tend to rely more on standard strategies and less often use shortcut strategies than boys (Hickendorff et al., 2010, 2018; Timmermans et al., 2007), but these results are not consistent across studies (Torbeyns et al., 2017).

Most of these studies in which the relation between student factors and students' strategy use was investigated focused on practical flexibility and practical task-based adaptivity only, and findings were at times inconsistent. The current study therefore aims to broaden the scope and the insights on the relation between student factors and students' adaptive expertise, by investigating the role of mathematics achievement level, adaptive number knowledge, working memory, and gender in the practical and potential components of the three constructs of adaptive expertise.

Current Study

The current study aimed to not only chart the practical adaptive expertise that third graders show in the domain of multi-digit subtraction, but also to investigate their (hidden) potential. For each of the three constructs of adaptive expertise – flexibility, task-based adaptivity, and individual-based adaptivity – two measures were created: the first addressing the practical component (what students show) and the second addressing the potential component (what students know). The study was guided by four research questions:

1. To what extent do students use different strategies (practical flexibility) and to what extent do they know different strategies (potential flexibility)?
2. To what extent do students use strategies that are optimal for the task (practical task-based adaptivity), and to what extent do they know which strategy is optimal for the task (potential task-based adaptivity)?
3. To what extent do students use strategies that are optimal for themselves (practical individual-based adaptivity), and to what extent do they know which strategy is optimal for themselves (potential individual-based adaptivity)?
4. How are the six constructs of the adaptive expertise framework related to students' mathematics achievement level, adaptive number knowledge, working memory capacity, and gender?

Method

Participants

Participants were 147 third graders (49.7% boys) from 12 primary schools from the Netherlands. The research protocol was approved by the Institute's IRB [number ECPW2019-242] and only children with written parental consent and individual consent participated. As an indicator of mathematics achievement level, we collected the students' most recent score on the standardized mathematics subtest of CITO's student monitoring system, which was usually the end of grade 2 assessment (Janssen et al., 2015). Scores are divided in five norm-referenced quantiles. There were 55 students (37.9%) in the highest Quantile I, 33 students (22.8%) in Quantile II, 36 students (24.8%) in Quantile III, 10 students (6.9%) in Quantile IV, and 11 students (7.6%) in the lowest Quantile V (missing data for 2 students). In all, the sample thus overrepresented high mathematics achievers and underrepresented low mathematics achievers, compared to the Dutch population.

Materials and Procedure

Arithmetic Task

The arithmetic task consisted of four tasks, administered in two sessions. Tasks 1-3 were administered in classroom setting in Session 1 in maximally 60 minutes, Task 4 was administered individually in Session 2 in maximally 20 minutes.

All tasks consisted of subtraction problems up to 100. We distinguished four problem types, based on the number characteristics that fitted one of the four main strategies jump, split, compensation and indirect addition. Jump problems had a large difference (> 40) and involved crossing tens, e.g., 63 − 17. Split problems had a large difference but did not require crossing tens, e.g., 84 − 32. Compensation problems also had a large difference, and the units of the minuend were 8 or 9, e.g., 68 − 19. Finally, indirect addition problems had a small difference (< 10) and involved crossing the tens, e.g., 82 − 76.

Task 1 and 2 focused on flexibility and included all four problem types. Tasks 3 and 4 focused on adaptivity and therefore included only problems suited for the task-specific strategies compensation and indirect addition. Table 2 gives an overview of how each task was used to operationalize the main constructs of the adaptive expertise framework.

Table 2

Overview of Operationalization of the Different Constructs

Construct	Task	Description
Practical flexibility	1a	The number of different strategies used to solve eight subtraction problems (one attempt)
Potential flexibility	1, 2	The number of different strategies used to solve eight subtraction problems (three attempts), plus the number of additional strategies completed correctly in Task 2
Practical task-based adaptivity	1a	The number of task-specific strategies used on the four problems suited for task-specific strategies
Potential task-based adaptivity	3	The number of correctly selected task-specific strategies from four strategy options
Practical individual-based adaptivity	1a, 4a	The number of optimal strategies used in Task 1a, based on individual speed and accuracy in Task 4a
Potential individual-based adaptivity	4	The number of correctly identified optimal strategies in Task 4b, based on individual speed and accuracy in Task 4a

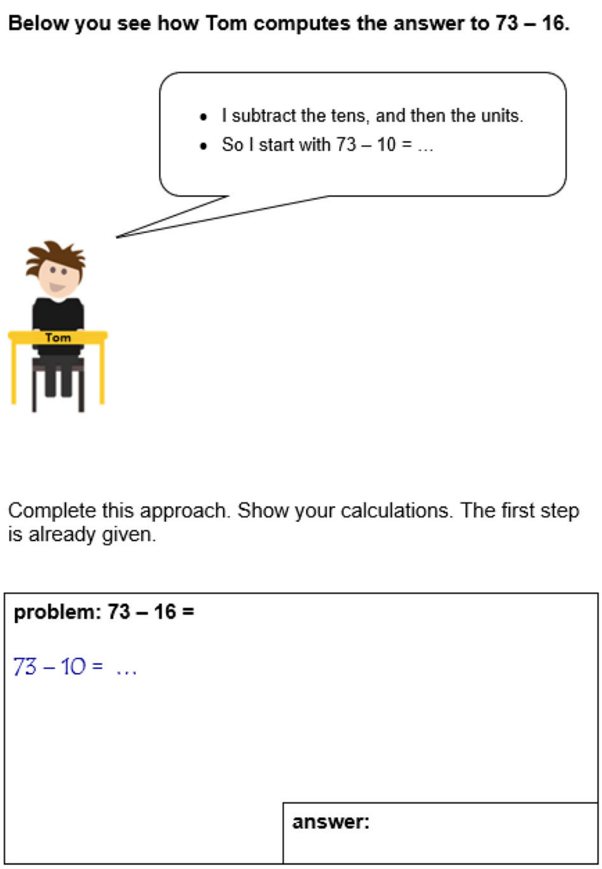
Task 1 consisted of eight subtraction problems (two per strategy) with three calculation boxes per problem. Students were instructed to solve these problems with the strategy they preferred. They had to write the solution steps in the upper calculation box (Task 1a). After they had completed all eight problems, they had to solve the same eight problems with different strategies than in their first attempt, for which they could use the two additional calculation boxes (Task 1b).

Strategies were coded into one of seven categories: jump, split, compensation, indirect addition, split-jump (combination of split and jump strategy; e.g., 84 − 67 via 80 − 60 = 20; 24 − 7 = 17), simplifying (changing both minuend and subtrahend to simplify the problem; e.g., 84 − 67 via adding 3 to both operands and solving 87 − 70 = 17), or indirect subtraction (computing how much has to be subtracted from the minuend to reach the subtrahend; e.g., 84 − 67 via 84 − 10 = 74; 74 − 7 = 67, so the answer is 10 + 7 = 17). For a random selection of 20% of the students (n = 29) the strategies on the first attempt were coded by a second, independent, researcher. The two raters agreed on 87% of the 232 solutions, with Cohen's kappa = .783 (substantial to good inter-rater reliability).

Task 2 consisted of four problems (one per strategy) that were presented with a picture of a child solving that problem with the strategy most suited for the problem (see example in Figure 1). The first solution step was given, and students were asked to complete that strategy. The written calculation steps and the answer that the students wrote down were used to code whether the intended strategy was completed correctly.

Figure 1

The Jump Problem From Task 2

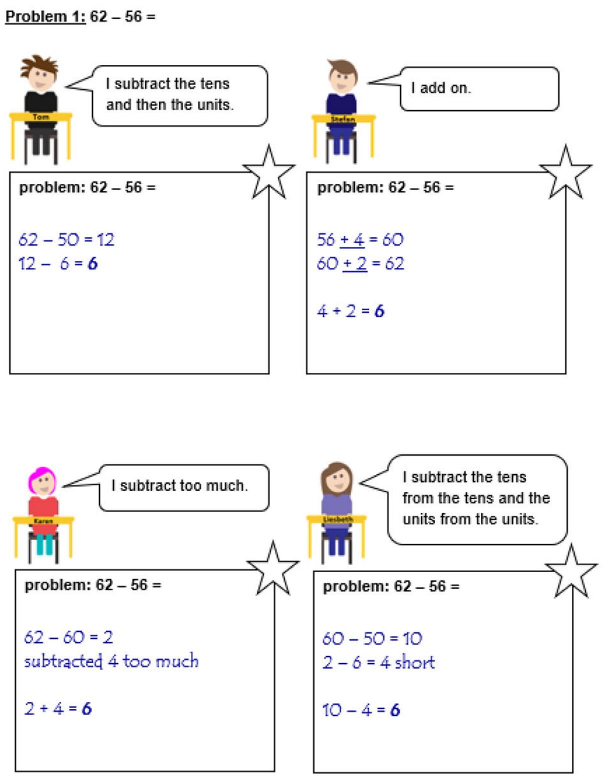


In Task 3 students were offered four problems (two for each of the task-specific strategies compensation and indirect addition). Each problem was solved by four different children each using one of the four main strategies. Students were asked to select the optimal strategy for the problem (Figure 2).

In Task 4 students solves the two compensation problems and two indirect addition problems from Task 1 again. Task 4a was a personalized version of the no-choice conditions of the choice/no-choice design (Siegler & Lemaire, 1997). Students had to solve these four problems twice: once with the task-specific strategy (i.e., compensation on the compensation problems and indirect addition on the indirect addition problems) and once with the strategy they had themselves used on these problems in Task 1a. If they had used the task-specific strategy spontaneously in Task 1a, or if the strategy in Task 1a was not classifiable, the strategy that they had to use in the second no-choice condition was determined as the strategy they used most frequently on the other seven problems in Task 1a. For instance, a student who solved 86 – 29 in Task 1a with the jump strategy had to solve this problem with the compensation strategy in the first no-choice condition and with the jump strategy in the second no-choice condition. A student who solved problem 64 – 57 in Task 1a with the indirect addition strategy had to solve this problem with indirect addition in the first no-choice condition and with the strategy (s)he used most frequently on the other seven problems in Task 1a in the second no-choice condition.

Figure 2

Example Problem From Task 3



Strategy instruction cards and verbal instructions were used to instruct students which strategy they had to use. For each solution attempt, the experimenter recorded the solution time and scored the answer as correct or incorrect. Those speed and accuracy data were used to determine which strategy was optimal, i.e., the fastest strategy leading to a correct answer. This information was used to determine whether the strategy used in Task 1a was the optimal one for that particular student on that particular problem (practical individual-based adaptivity). Finally, in Task 4b students were shown their two no-choice solutions from Task 4a and had to select the strategy they thought was optimal for them. This captured whether they knew which of the two strategies they completed was optimal (potential individual-based adaptivity).

Adaptive Number Knowledge

To measure students' adaptive number knowledge we used the Arithmetic Production Task (McMullen et al., 2016). This is a timed, paper-and-pencil instrument that aims to capture students' ability to recognize and use different numerical characteristics and relations. Each item consists of five given numbers and the four basic operations addition, subtraction, multiplication, and division. With these numbers and operations, students must produce as many arithmetic sentences as possible that equal a given target number, in 90 seconds. There was one practice item followed by four items that students had to answer independently: (a) target number 16 with given numbers 2, 4, 8, 12, and 32; (b) target number 18 with given numbers 2, 3, 9, 15, and 36; (c) target number 12 with given numbers 2, 4, 6, 16, and 24; and (d) target number 24 with given numbers 2, 4, 6, 16, and 48. The total number of correct arithmetic sentences across the four items was used as a measure of adaptive number knowledge. This task was administered in Session 1 (classroom setting), after arithmetic Tasks 1-3, in maximally 10 minutes.

Working Memory

The forward and backward digit-span tasks from the WISC-III (Wechsler, 1991) were administered. The researcher read out series of digits, which students had to recall in the same order (forward task) or in the opposite order (backward task). The series increased in length from two to nine digits, with two trials per length. When both trials of a certain length were answered incorrectly the task was stopped. The total number of correctly recalled series in the backward digit span task was used as a measure of students' working memory capacity. This task was administered in Session 2 (individual testing), after arithmetic Task 4, in maximally 5 minutes.

Results

Research Question 1: Flexibility

In the first attempt to solve the eight problems of Task 1, 60% of the solutions involved the jump strategy, 21% involved the split strategy, and 9% involved the split-jump strategy. The other strategies were used very rarely. This did not change much when students solved the same problems for a second and third time.

Table 3 shows the distribution of the number of different strategies students used across the eight problems, on the first attempt and on the first to third attempt combined. On the first attempt, a large majority (80%) of the students consistently used the same strategy on all eight problems, which was most often the jump strategy (69 students) or the split strategy (24 students). A small number of students used two (12%) or three (4%) different strategies across the problems. For the remaining five students, all eight solutions fell into the categories unclear strategy, without written work and/or problem skipped. When the solutions on the first to third attempt were combined, still a majority (64%) used only one strategy on all attempts. Students used on average more different strategies when they had three attempts (M = 1.40) compared to one attempt (M = 1.17); $t(146) = 6.172$, $p < .001$. Analyzed per problem, 14-20% of the students used at least two different strategies in the three attempts. The number of different strategies used on the first attempt across the eight problems was used as the measure of *practical flexibility* (see also Table 4 for the descriptive statistics of all constructs).

Table 3

Distribution of Number of Different Strategies Used Across All Problems in Task 1

Number of different strategies	Attempt 1	Attempts 1-3 combined
0	5 (3%)	3 (2%)
1	118 (80%)	94 (64%)
2	18 (12%)	39 (27%)
3	6 (4%)	10 (7%)
4	0 (0%)	1 (1%)
M (*SD*)	1.17 (0.541)	1.40 (0.679)

Potential flexibility was conceptualized as the number of strategies students know, and comprised two parts. The first part was the number of different strategies used across all three attempts in Task 1. The second part was based on Task 2, where students had to complete the four main strategies. For each of these strategies, one point was given if a strategy that was not part of the student's strategy repertoire across all three attempts in Task 1 was completed correctly in Task 2. There were 13 students with hidden potential for the jump strategy, 13 for the split strategy, 24 for the compensation strategy and 12 for the indirect addition strategy. On average students knew 0.42 more strategies than they used in Task 1. A composite measure of *potential flexibility* was constructed by summing these two parts (M =

1.82). The mean *potential* flexibility was significantly higher than the mean *practical* flexibility, $t(146) = 10.743$, $p < .001$, suggesting hidden potential in strategy knowledge.

Table 4

Descriptive Statistics and Inter-Correlations of the Six Adaptive Expertise Constructs, Mathematics Achievement Level, Adaptive Number Knowledge, and Working Memory

Construct		*M*	*SD*	*N*	1	2	3	4	5	6	7	8
1	Pr Flex	1.17	0.541	147								
2	Pot Flex	1.82	0.897	147	.57**							
3	Pr Ad (t)	0.07	0.372	147	.45**	.25**						
4	Pot Ad (t)	0.71	0.878	147	.09	-.01	-.08					
5	Pr Ad (i)	0.79	2.070	117	-.06	-.13	-.04	-.03				
6	Pot Ad (i)	0.53	2.182	119	.07	.14	.09	-.12	.26**			
7	ML	3.77	1.242	145	.21*	.40**	.16	.00	.03	.22*		
8	ANK	12.83	4.863	143	.11	.26**	.06	.02	-.15	-.02	.51**	
9	WM	4.05	1.391	142	.11	.20*	-.01	-.01	-.16	.03	.21*	.27**

Note. Pr Flex = Practical flexibility; Pot flex = Potential flexibility; Pr Ad (t) = Practical task-based adaptivity; Pot Ad (t) = Potential task-based adaptivity; Pr Ad (i) = Practical individual-based adaptivity; Pot Ad (i) = Potential individual-based adaptivity; ML = Mathematics achievement level; ANK = Adaptive number knowledge; WM = Working memory.
*$p < .05$. **$p < .01$.

Research Question 2: Task-Based Adaptivity

The number of times the two indirect addition problems and the two compensation problems were solved with their respective task-specific strategies in Task 1a was very low: 140 students (95.2%) never used the task-specific strategy, the other seven students used it on one to three problems. The measure of *practical task-based adaptivity* was the number of problems solved with the task-specific strategy ($M = 0.07$).

In Task 3, 79 students (53.7%) never selected the task-specific strategy as the optimal one across the four problems, and the remaining 68 students made one to three adaptive strategy selections. The measure of *potential task-based adaptivity* was the number of problems for which the adaptive, task-specific strategy was selected ($M = 0.71$). The mean *potential* task-based adaptivity was significantly larger than the mean *practical* adaptivity, $t(146) = 7.829$, $p < .001$, indicating hidden potential in adaptivity to task characteristics.

Research Question 3: Individual-Based Adaptivity

To determine the optimal strategy for each student on each problem, we compared the accuracy and speed of the task-specific and the other strategy in Task 4a to determine the fastest strategy leading to a correct answer, in a similar way as in previous studies (Fagginger Auer et al., 2016; Hickendorff, 2020). First, if one strategy yielded the correct answer and the other did not, the correct strategy was coded as the optimal one (24% of all solutions). If both strategies yielded incorrect answers, it was not possible to determine the optimal strategy (4% of all solutions). If both strategies yielded the correct answer we turned to speed and coded the faster strategy as the optimal strategy (55% of all solutions). Cases in which students did not use the strategy they were supposed to use in one or both no-choice conditions were excluded.

The indirect addition strategy was the optimal strategy in 19% of the indirect addition problems, and the compensation strategy in 39% of compensation problems. The other, non-task-specific strategy that students themselves had used in Task 1a, was optimal in 60% of the indirect addition problems and in 39% of the compensation problems.

Practical Individual-Based Adaptivity

For each student we counted the number of times the optimal strategy as determined from Task 4a was the strategy used in Task 1a. Eleven students (7.5%) never used their optimal strategy, and 106 students (72.1%) used it one to four times. The remaining 30 students had a missing value because they did not use the intended strategies in the no-choice conditions, or they did not use one of the no-choice strategies in Task 1a. On average, students used their optimal strategy on 2.13 problems and used the non-optimal strategy (i.e., the incorrect strategy, or the slower strategy when both strategies yielded the correct answer) on 1.33 problems, a significant difference, $t(116) = 4.153$, $p < .001$. As the measure of *practical individual-based adaptivity* we computed a “net” score of the number of adaptive strategy choices: for all instances where it was possible to determine which of the two strategies was optimal, one point was given when the optimal strategy was used in Task 1a, but one point was subtracted when the other, non-optimal strategy was used in Task 1a. On average this lead to a positive average number of individual-based adaptive strategy choices ($M = 0.79$).

Potential Individual-Based Adaptivity

For each student we counted the number of times the optimal strategy as determined from Task 4a was the strategy students selected as they thought was optimal in Task 4b. Fourteen students (9.5%) never selected their optimal strategy, 116 students (78.9%) selected it one to four times. The remaining 17 students had a missing value because they had two or more missing values on the optimal strategy determination. On average, students selected their optimal strategy on 2.01 problems and selected the non-optimal strategy on 1.48 problems, a significant difference, $t(129) = 2.843$, $p = .005$. As a measure of *potential individual-based adaptivity* we again computed a “net” score of adaptive strategy choices by giving credit points for selecting the optimal strategy and penalty points for selecting the other, non-optimal strategy ($M = 0.53$). The mean *potential* individual-based adaptivity did not differ from the mean *practical* adaptivity, $t(115) = 1.367$, $p = .17$.

Research Question 4: Relation With Individual Differences

Correlation analyses were used to address the relation between the six adaptive expertise constructs on the one hand and students’ mathematics achievement level, adaptive number knowledge, and working memory on the other (see Table 4). In general there were only few significant relations. Mathematics achievement level was significantly correlated with three of the six constructs: practical flexibility ($r = .21$), potential flexibility ($r = .40$), and potential individual-based adaptivity ($r = .22$). Both adaptive number knowledge and working memory were related to only one adaptive expertise construct: potential flexibility ($r = .26$ and $r = .20$, respectively). To investigate gender differences, a multivariate ANOVA with the six adaptive expertise constructs as dependent variables and gender as factor showed there were no significant differences between boys and girls: multivariate $F(6, 109) = 0.697$, $p = .653$. In all, potential flexibility was related to all student factors except gender, and practical flexibility and potential individual-based adaptivity were related to mathematics achievement level. The three remaining adaptive expertise constructs, practical task-based adaptivity, potential task-based adaptivity, and practical individual-based adaptivity, were unrelated to all student factors investigated.

Discussion

The current study aimed to provide a comprehensive investigation of third graders’ adaptive expertise in the domain of multidigit subtraction, focusing on three main constructs – flexibility, task-based adaptivity, and individual-based adaptivity – and distinguishing between a practical component (what students show) and a potential component (what students know) for each construct.

Students’ level of practical flexibility and practical task-based adaptivity was rather low, which is in line with previous research (De Smedt et al., 2010; Hickendorff et al., 2018; Selter, 2001; Torbeyns et al., 2017). Many students consistently used the same strategy across problems (usually jump or split) and hardly ever used the task-specific strategies indirect addition and compensation, not even on the problems with number characteristics intended to elicit those strategies. However, students do seem to know more strategies (potential flexibility) than they spontaneously use (practical flexibility). When prompted to solve the same problem in different ways students used more different strategies than on their first attempt, as in Blöte et al. (2001), Newton et al. (2020), Torbeyns et al. (2009a), and Xu et al. (2017), although in these studies more students were able to produce different strategies than in the current study.

Similarly, students' potential task-based adaptivity was higher than their practical adaptivity: nearly half of the students could select the task-specific shortcut strategy (indirect addition or compensation) from four given strategies at least once, similar to findings for secondary students' equation solving (Newton et al., 2020; Xu et al., 2017).

To address individual-based adaptivity a personalized version of the choice/no-choice method was used (Hickendorff, 2020; Siegler & Lemaire, 1997). Instead of focusing on two pre-selected strategies, one pre-defined strategy (the task-specific shortcut strategy) was compared to the students' own preferential strategy, which could vary between students and between problems within students. This had three advantages. First, the choice condition involved free strategy choice instead of restricted choice, enabling to assess students' strategy repertoire and selection processes in a more complete and more naturalistic way (Luwel et al., 2009). Indeed, there was large between-student variability in the subtraction strategies students used. Second, it allowed comparing the performance of the task-specific strategy with the performance of students' own preferential strategy. Findings showed that the task-specific strategy was optimal less often than students' own strategy. This provides empirical evidence for the statement that shortcut strategies are not necessarily *easy* strategies for all students, since they require good understanding of numbers and relations (Hickendorff, 2018; Torbeyns et al., 2009b). Third, it provides the opportunity to address individual-based adaptivity in a more complete and more ecologically valid way, by focusing on the strategies students used spontaneously in the free choice condition. Research addressing adaptivity by focusing solely on the use of task-specific strategies thus provides a one-sided picture of adaptivity: students might not use shortcut strategies since for them, they do not work well. In the current study the level of individual-based adaptivity was indeed higher than the level of task-based adaptivity.

Taken together, these findings imply on the one hand that the task-specific strategy, although it has the fewest and the easiest steps, is usually not optimal for an individual student. On the other hand, there is a discrepancy between the very low frequency of spontaneous use of the task-specific strategies (1-4%) and the percentage of trials where the task-specific strategy is optimal for an individual student (19-39%). This discrepancy suggests hidden potential for the adaptive use of task-specific strategies, particularly for the compensation strategy. However, although students on average knew which strategy was optimal for them, their potential individual-based adaptivity did not differ from their practical individual-based adaptivity, so there was no indication of hidden potential in individual-based adaptivity.

Finally, we addressed the relation between mathematics achievement level, adaptive number knowledge, working memory, and gender on the one hand, and the six components of adaptive expertise on the other. Mathematics achievement level was positively related to both practical and potential flexibility, as well as to potential individual-based adaptivity. This is in line with earlier findings that students with higher mathematics level show more strategy variability (Hickendorff et al., 2010; Newton et al., 2020; Torbeyns et al., 2006, 2017, 2018) but also extends this relation to other components of adaptive expertise. Interestingly, Newton et al. (2020) also found that students' prior knowledge was related to whether students recognized multiple solution strategies, which is similar to the potential flexibility measure in the current study. Importantly, in the current study students' potential flexibility was found to be related not only to general mathematics achievement level but also to more specific mathematical knowledge (adaptive number knowledge) as well as to domain-general cognitive capacity (working memory). As such, knowing multiple strategies may be related to general intellectual capacity rather than domain-specific knowledge.

Unexpectedly, all other measures of adaptivity were unrelated to mathematics achievement, adaptive number knowledge, or working memory. For practical task-based adaptivity the absence of significant relations can probably be explained by a floor effect, given the low frequency with which the task-specific shortcut strategies were used spontaneously. For individual-based adaptivity it is important to note that the current study was the first to compare the task-specific strategy to students' own preferential strategy. That is, if students are competent in their preferential strategy, such as the jump or split strategy, but not in the task-specific strategy, they can score high on individual-based adaptivity by consistently using their own strategy. This could possibly explain that for this type of adaptivity, mathematical knowledge and domain-general cognitive capacities do not play a prominent role.

Finally, there were no gender differences in the six components of adaptive expertise. Previous studies show inconsistent findings. Studies that have found gender differences show that girls tend to rely more on standard strategies and less often use shortcut strategies than boys (Hickendorff et al., 2010, 2018; Timmermans et al., 2007). Again, perhaps the low base rate of using multiple strategies and of using task-specific strategies explained the absence of gender differences.

Educational Implications

Although there is hidden potential in the components of adaptive expertise, even when students would fully exploit this potential, their level of flexibility and adaptivity would still be rather low in an absolute sense and also compared to other studies capturing potential flexibility. A salient question is therefore how to raise the level of adaptive expertise. Some instructional factors seem to have a positive impact on children's flexibility and adaptivity: stimulating children to invent, reflect, and discuss strategies, compare worked-out examples of different strategies presented side-by-side, and teachers asking open questions (Blöte et al., 2001; Heinze et al., 2018; Star et al., 2015), but rigorous studies are scarce. Recently, German mathematics textbooks were found to differ in the quality of learning opportunities for adaptive expertise, and these differences were related to students' adaptive expertise after three years of schooling (Sievert et al., 2019). Future studies should address the classroom practices and learning resources simultaneously to give insights into promising approaches to foster students' adaptive expertise at different curriculum levels.

Looking at the types of strategies, previous research has already shown that primary school students can apply the indirect addition strategy efficiently and adaptively after a brief instruction (Hickendorff, 2020; Torbeyns et al., 2018). The current study's findings suggest that there is particularly hidden potential for the compensation strategy. Heinze et al. (2018) showed that students can self-invent the compensation strategy without explicit instruction. Thus, fostering the use of the compensation strategy by providing well-chosen problems and stimulating discussion could potentially raise adaptive strategy choices. Finally, noteworthy is the high frequency of the split strategy, which is error-prone in subtraction. Discouraging the use of the split strategy could perhaps also foster adaptive strategy choices.

Methodological Considerations

Several methodological issues and limitations are important to discuss. First, the measure of potential flexibility comprised two components: prompting students to use a different strategy on the same problem (as was done in previous studies) and prompting students to complete a particular strategy, where the first step and a general verbal description of the strategy were given. This second approach has not been used before, to our knowledge. It is arguably different than the first approach, since part of the solution strategy is given away. It could be that students produced these alternative strategies for the very first time given this prompt. But even if this were so, this would mean that a single hint could lead students to use a strategy they did not used before, and thus that this strategy knowledge is very much at the surface and as such does give insights into their potential flexibility. Future attempts to measure potential flexibility in mathematics could add a similar measure to their instruments to investigate this issue further.

Second, by focusing on the number domain up to 100, the structural features of the problems intended to elicit the task-specific shortcut strategies compensation and indirect addition might not have been very salient. That is, in subtraction op to 1000 more solution steps have to be taken, making the benefits of rounding the subtrahend in for instance $743 - 399$ or adding on in for instance $602 - 596$ more salient. Future research could extend the domain to subtraction up to 1000 to gain more insight into the balance between benefits of task-based shortcut strategies and students' individual strategy performance with these strategies in students' strategy choices.

Third, a drawback of the implemented choice/no-design is that students solved identical problems in two different tasks (free choice and no-choice conditions), which could lead to retesting effects. However, because the two sessions were at least one week apart and subtraction problems are a frequent topic in mathematics textbooks (Van Zanten & van den Heuvel-Panhuizen, 2014), we expect it is unlikely that students recognized the problems and remembered their solution. Furthermore, students had to solve the same problem twice within one session in the no-choice conditions. Since they had to use different strategies, we believe that it is unlikely that they could utilize their solution from one no-choice condition in the other.

Fourth, another weakness related to the choice/no-choice design is that it is necessary that strategies are identifiable on a problem-by-problem basis (Luwel et al., 2009; Siegler & Lemaire, 1997). We made this possible by requiring students to write down their calculation steps, which could affect their solution process (Hickendorff et al., 2010). Furthermore, there was no perfect agreement between the two raters in strategy classification. Since strategy classification determined how students had to solve the problems in the no-choice conditions, unreliability could affect the results regarding individual-based adaptivity. We argue that for analyses at the group level these classification errors probably cancel each other out, therefore not affecting the overall picture of patterns. However, for the diagnosis of individual students' adaptive strategy choices a more reliable classification system is necessary.

Finally, the study has a cognitive-psychological perspective on adaptive expertise and how to foster it, but affective factors and the socio-cultural context are also important (Verschaffel et al., 2009). Students develop (implicit) knowledge regarding the sociocultural norms – what is valued as appropriate, adaptive, and wise – which affects their strategy choices (Ellis, 1997). Future research should take that into account as well.

Conclusion

The current study showed that students' strategy flexibility and task-based adaptivity is rather low. However, including individual-based adaptivity (i.e., which strategy is optimal for an individual student) as well as students' potential in flexibility and adaptivity presents a more nuanced, and also more positive, picture of students' adaptive expertise. Students might not use task-specific strategies because these strategies do not work well for them. Furthermore, many students know more than they show regarding flexibility and task-based adaptivity.

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