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Reasoning About Fraction and Decimal Magnitudes, Reasoning Proportionally, and Mathematics Achievement in Australia and the United States

Ilyse Resnick[1], Nora Newcombe[2], Micah Goldwater[3]

[1] *STEM Education Resource Centre, Faculty of Education, University of Canberra, ACT, Canberra, Australia.* **[2]** *Department of Psychology, Temple University, Philadelphia, PA, USA.* **[3]** *School of Psychology, University of Sydney, Sydney, Australia.*

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Corresponding Author: Ilyse Resnick, 170 Haydon Drive, Bruce, ACT, Australia, 2617. E-mail: ilyse.resnick@canberra.edu.au

Abstract

There is strong evidence from research conducted in the United States that fraction magnitude understanding supports mathematics achievement. Unfortunately, there has been little research that examines if this relation is present across educational contexts with different approaches to teaching fractions. The current study compared fourth and sixth grade students from two countries which differ in their approach to teaching fractions: Australia and the United States. We gathered data on fraction and decimal magnitude understanding, proportional reasoning, and a standardized mathematics achievement test on whole number computation. Across both countries, reasoning about rational magnitude (either fraction or decimal) was predictive of whole number computation, supporting the central role of rational number learning. However, the precise relation varied, indicating that cross-national differences in rational number instruction can influence the nature of the relation between understanding fraction and decimal magnitude and mathematics achievement. The relation between proportional reasoning and whole number computation was fully mediated by rational magnitude understanding, suggesting that a key mechanism for how reasoning about rational magnitude supports mathematics achievement: proportional reasoning supports the development of an accurate spatial representation of magnitude that can be flexibly and proportionally scaled, which in turn supports children's mathematics learning. Together, these findings support using measurement models and spatial scaling strategies when teaching fractions and decimals.

Keywords

fractions, decimals, magnitude representation, proportional reasoning, cross-national comparison, mathematics achievement

Being able to reason about fraction and decimal magnitudes is essential for progress in mathematics (National Mathematics Advisory Panel, 2008). Fraction and decimal magnitude understanding are highly predictive of algebra knowledge specifically (e.g., Booth & Newton, 2012; Booth et al., 2014; DeWolf et al., 2015) and concurrent and later mathematics achievement more broadly (e.g., Bailey et al., 2012; Resnick et al., 2016; Siegler et al., 2012).

Unfortunately, most research on the predictive role of magnitude understanding has taken place in the United States, limiting generalizability to other cultures and educational contexts (with exception of Liu, 2018; Siegler et al., 2012; Torbeyns et al., 2015). The present study examines the role of fraction and decimal magnitude understanding in general mathematics achievement for students in two Western, English-speaking countries (Australia and the United States) that differ in their approach to teaching fractions and the timing of decimals instruction. In addition, the present study explores the role of proportional reasoning in supporting mathematics achievement via rational magnitude understanding (i.e., having an accurate spatial representation of magnitude that can be flexibly and proportionally scaled).

Research on the Relations Between Fraction and Decimal Magnitude Understanding and Mathematics Achievement Within the United States

In the United States, there is growing evidence of a strong, consistent, and predictive relation between understanding fraction magnitude and mathematics achievement. Cross-lagged panel models demonstrate longitudinal stability of this relation across fourth through seventh grade (Bailey et al., 2012; Hansen et al., 2017). Growth in fraction magnitude understanding over the intermediary grades is predictive of later sixth grade mathematics achievement (Resnick et al., 2016). From sixth through eighth grade, after the bulk of fraction instruction is completed, the gap between students with a low versus high understanding of fraction magnitude continues to widen, and the relation between fraction magnitude understanding and mathematics achievement strengthens (Siegler & Pyke, 2013).

Extending to decimal magnitude, Resnick et al. (2019) found that reasoning about fraction, decimal, and whole number magnitude (assessed at the beginning of fourth grade) all contributed unique variance when predicting later mathematics achievement at the end of fourth grade, even after controlling for a range of behavioral and cognitive factors. In contrast, DeWolf et al. (2015) found concurrent seventh grade algebra knowledge related only to reasoning about decimal magnitude and fraction relations specifically (assessment items focused on the relation between the numerator and denominator, e.g., inverse relations and equivalence, and not on reasoning about magnitude). Reasoning about fraction and whole number magnitude and understanding procedural fraction knowledge were not significant predictors. Although DeWolf et al. (2015) suggested that reasoning about decimal magnitude involves a "purer" representation of magnitude compared to fractions, the different findings may be due to young children using rule-based reasoning whereas older children have access to a wider range of strategies including analog representations (Resnick et al., 2019).

Research on the Relations Between Fraction and Decimal Magnitude Understanding and Mathematics Achievement Outside of the United States

To our knowledge, there are no studies that have examined the role of decimal understanding supporting mathematics achievement outside of the United States. However, two studies have examined the role of fraction magnitude understanding (Liu, 2018; Torbeyns et al., 2015), and one study examined general fraction knowledge more broadly (Siegler et al., 2012). General fraction knowledge can include both procedural (e.g., fraction addition) and conceptual (e.g., fair sharing) understanding, with fraction magnitude being just one kind of conceptual knowledge. In Siegler et al.'s (2012) study, fraction knowledge was assessed using fraction items from either the British Cohort Study's Friendly Maths Test (Butler & Bynner, 1980, 1986; Bynner et al., 1997) or the Calculation subtest of the Woodcock-Johnson Psycho-Educational Battery − Revised (WJ-R) for the United Kingdom (n = 3,677) and United States (n = 599) samples, respectively. Performance at 10 years of age on the fraction items predicted high school mathematics achievement for both nationally representative samples even after controlling for mathematical knowledge, general intellectual ability, working memory, and family income and education.

Liu (2018) conducted a cross-sectional study of students in China prior to primary fraction instruction (beginning of fourth grade; n = 35) and after a year of primary fraction instruction (end of fourth grade; n = 40). Fraction magnitude understanding was assessed using a fraction number line estimation task; students were asked to identify where given fractions would be located on a 0-1 number line. When controlling for whole number skills, fraction magnitude understanding predicted mathematics achievement after primary fraction instruction but not prior. Given that no fraction estimation strategy was observed in approximately 50% of this sample, one explanation for not observing a relation prior to fraction instruction is that many of these students were likely not actually reasoning about fraction magnitude.

Torbeyns et al. (2015) is the only study to consider cross-national differences in teaching fraction magnitude and the relation of teaching practices to overall mathematics achievement (see section below on cross-national differences in teaching fractions). Sixth and eighth grade students from China, Belgium, and the United States completed the following tasks: fraction number estimation (0-1, 0-5), fraction magnitude comparison (i.e., determining which of two fractions is larger), fraction arithmetic (addition, division), and a country-specific general mathematics achievement test. Across all three countries, a composite fraction magnitude estimation score (mean of both number lines) was predictive of concurrent mathematics achievement after controlling for fraction arithmetic. China and Belgium were more accurate compared with the United States on the fraction number line estimation and fraction comparison. The authors pointed to differences in educators' fraction knowledge, instructional tools, and focus on teaching fractions as magnitudes compared with a focus on fractions as a part-whole relation.

Cross-National Differences in Mathematics Education

International comparisons of mathematics education can characterize differences in state or national standards, teacher's content knowledge, preparation, and instructional practices, and their relation to student outcomes. Torbeyns et al. (2015) found that China, Belgium, and the United States differed in all these areas. China, Belgium, and the United States also have many cultural differences (e.g., Eastern vs. Western cultures, language, parental beliefs about school readiness) that may influence student mathematics outcomes (e.g., Miller et al., 2005). Thus, it is difficult to identify how teaching practices specifically support the relation between fraction magnitude understanding and mathematics achievement in these countries.

The current study focuses on two countries that are more similar: Australia and the United States. Both countries are primarily Western and English speaking. Primary school educators from Australia and the United States have similar rates of completing a mathematics degree (13%), completing at least a bachelor's degree (93-100%), years of experience (average 13-15 years), and having undertaken professional learning in mathematics content and pedagogy in the past two years (Fishbein et al., 2021). Studies on pre-service education have highlighted difficulties for educators in both countries understanding general mathematics content knowledge (e.g., Callingham et al., 2012; Senk et al., 2012) and fractions knowledge (for review see Olanoff et al., 2014). While the United States scored significantly higher in mathematics compared to Australia in the last cycle of the Trends in International Mathematics and Science Study (TIMSS), both countries scored lower than China and Belgium (Fishbein et al., 2021), and were ranked similarly on the Programme for International Student Assessment (PISA; OECD, 2019).

At the time of this study, the sequence and progression of fraction and decimal instruction described in state and national standards is also similar between Australia and the United States. The bulk of fraction instruction in both countries occurs over third through fifth grade, with foundational fraction concepts (e.g., partitioning and understanding "half") beginning in first grade (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; NSW Education Standards Authority, 2019). The overarching aims of these years align in both countries (e.g., a key outcome in fourth grade is understanding fraction equivalence). Decimal instruction is also similar in both countries' state and national standards: they begin by focusing on understanding decimals in terms of place value and in relation to fractions and percentages, with initial decimal instruction occurring in fourth grade (ACARA, 2014; NSW Education Standards Authority, 2019).

Importantly, Australia and the United States differ on their use of measurement models to teach fractions. The United States emphasizes, almost exclusively, a part-whole interpretation (Torbeyns et al., 2015); that is, equal partitioning of objects (e.g., cutting an apple pie into eight equal parts). This is in contrast with Australia, where state standards emphasize reasoning about a range of fraction representations, including linear, from as early as first grade (ACARA, 2014; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; NSW Education Standards Authority, 2019). A linear representation of fractions is consistent with a measurement model, where fractions represent values in between whole numbers (e.g., measuring lengths of objects with a standard ruler).

Measurement models have been identified as a key component in intervention studies for developing an accurate sense of fraction magnitude (e.g., Fuchs et al., 2013). For example, an intervention centering on learning fractions using a number line improved sixth graders' understanding of fraction magnitude and fraction concepts (Barbieri et al., 2020). In another intervention-based study, number line training transferred to second and third grader's performance on fraction magnitude comparison, whereas area models (i.e., part-whole models) did not (Hamdan & Gunderson, 2017). Notably, these intervention studies take place within the United States and outside normal class instruction.

Explanations for Why Fraction and Decimal Magnitude Understanding Supports Mathematics Achievement

Understood within overlapping waves theory (Siegler, 1996), mathematics learning may involve the flexible application of different sets of numerical rules that accurately represent numerical magnitude. That reasoning about whole number, fraction, and decimal magnitudes all uniquely predict later mathematics achievement (Resnick et al., 2019), suggests reasoning about magnitude is not a single skill, but rather dissociable based on type of number. Each type of number has different numerical properties. For example, multiplying two positive integers always increases its value, whereas multiplying two unit fractions always decreases its value (except for one and 1/1 where the value stays the same). Children use a range of strategies to reason about different types of magnitude based on their numerical properties, including the application of numerical rules (Resnick et al., 2019; Rinne et al., 2017) and the application of a linear representation of holistic magnitude (e.g., Schneider & Siegler, 2010).

It may be the case that a linear representation of magnitude supports mathematics achievement because it involves the ability to flexibly and proportionally scale mathematics information. Indeed, Matthews and Hubbard (2017) have identified spatial proportional reasoning as “underappreciated” in the literature and likely “critical” for the development of fractions knowledge and mathematics learning. Proportional reasoning involves understanding the multiplicative part-whole relations between rational quantities (Boyer & Levine, 2015). Matthews and Hubbard (2017) suggest that proportional reasoning may support the development of symbolic rational magnitude understanding, which in turn helps children reason about broader symbolic numerical tasks. For example, being able to proportionally reason about the location of numbers along a number line is related to many mathematics outcomes, such as number recall, approximate calculation, and symbolic estimation (Booth & Siegler, 2008; Laski & Siegler, 2007). Proportional reasoning also contributes unique variance to reasoning about fraction magnitude, even when controlling for spatial scaling, a separate but closely related spatial skill (Möhring et al., 2018). Spatial scaling involves relating distances in one space to distances in another space (Frick & Newcombe, 2012; Huttenlocher et al., 1999). Children with more fraction knowledge are less likely to be distracted by demarcations and are less affected by larger scale factors (Begolli et al., 2020), further highlighting the potential role of proportional reasoning and spatial scaling in magnitude understanding.

To date, we are not aware of any studies examining how magnitude understanding may mediate the relation between proportional reasoning and mathematics achievement. In Gunderson et al.’s (2012) seminal work, the mental number line was shown to fully mediate the relation between mental rotation and mathematics achievement, suggesting that spatial reasoning supports mathematics achievement by helping children create a linear spatial representation of magnitude. Similarly, Hawes et al. (2019) found that numerical ability (symbolic comparison, nonsymbolic comparison, and ordering) partially mediated the relation between spatial ability (mental rotation, visual-spatial reasoning, and Raven’s matrices) and mathematics performance (numeration and geometry). In contrast, Yang et al. (2021) did not find a mediative role of number line in connecting visual-perceptual and spatial visualization skills with arithmetic. The difference in findings between these studies may reflect the multidimensionality of spatial and mathematics skills, which may vary across development (Gunderson & Hildebrand, 2021).

Present Study

The present study examines the relation between fraction magnitude understanding and mathematics achievement in two countries – Australia and the United States – that differ on their use of measurement models in fraction education. This extends the work by Torbeyns et al. (2015) in several ways. The present study compares two countries that are similar across cultural factors, teacher content knowledge, and state/national standards, which helps clarify the role of measurement model versus part-whole model in fraction instruction. The present study also examines students in fourth and sixth grade, when the bulk of fraction instruction occurs, to examine if the relation between fraction magnitude understanding and mathematics achievement changes over the course of fraction instruction. Because Torbeyns et al. (2015) find similar relations between fraction number line estimation and fraction comparison with mathematics achievement, we extend this by replacing fraction comparison with decimal comparison to examine if representational format matters. In addition, we examine how proportional reasoning supports mathematics achievement by exploring mediative pathways via rational magnitude understanding. Aligned with previous work (Gunderson et al., 2012; Hawes et al., 2019), we hypothesize that the relation between proportional reasoning and mathematics achievement will be mediated by rational magnitude understanding. While the previous three studies conducted outside the United States used country-specific mathematics achievement tests, the present study uses the same standardized mathematics achievement test on whole number computation to allow for direct comparison.

Method

Participants

A sample of 156 students from Australia and the U.S. participated in this study. Information sheets and consent forms describing the study were sent home to all families with children in fourth or sixth grade (10 different classes) at three participating schools. The participating schools were private schools matched for demographics of the population served: religious affiliation, size, coed, tuition, and similar indexes of community socioeconomic advantage (e.g., the schools reported serving predominately white, English-speaking, middle to higher socioeconomic families). While each school used their own teacher-designed curriculum, all curricula were explicitly mapped onto their respective national education standards. The United States educators reported exclusively using part-whole models to teach fraction magnitude in fourth grade, in contrast with the Australian fourth grade educators reporting the inclusion of measurement models. In both countries the average age of students in fourth grade is nine to ten years old and in sixth grade is 11-12 years old. The Australian sample was comprised of 41 students in fourth grade and 34 students in sixth grade (45% female, 55% male), and the U.S. sample was comprised of 35 students in fourth grade and 46 students in sixth grade (48% female, 52% male).

Materials

Fraction Number Line Estimation

The same number line estimation tasks (0-1, 0-5) as Torbeyns et al. (2015) were used. Students were presented with a number line in the middle of a touchscreen tablet, with hatch marks and labels (i.e., 0, 1 or 5) at either end. Fractions were presented one at a time above the middle of the number line. Students touched their finger on the number line where they thought each fraction was located. The following fractions were used for the 0-1 number line (1/19, 1/9, 1/7, 1/4, 1/2, 3/8, 2/5, 4/9, 4/7, 5/8, 2/3, 7/9, 5/6, 7/8, 12/13) and the 0-5 number line (2/9, 1/2, 4/7, 5/3, 17/9, 2/1, 9/4, 19/8, 8/3, 13/4, 7/2, 23/6, 4/1, 22/5, 9/2). Accuracy was measured using percent absolute error (PAE), calculated as the absolute difference between the estimated and actual magnitudes divided by the numerical range of the number line (1 or 5), and then multiplying by 100 for each estimate. For example, if a child was asked to locate 2/9 on a 0–5 line and marked the location corresponding to 5/4 the PAE would be 20.55% [|(2/9 – 5/4)|/5*100]. Each student was assigned a single score by taking their mean PAE.

Decimal Comparison

A decimal comparison task was developed based on Resnick et al. (2019) to assess student's understanding of decimal magnitude. Students were asked to identify which of two decimals was larger. There were three types of trials, each with four comparison trials (12 total), which corresponded to common decimal biases (for details see Resnick et al., 2019). For each of the comparisons, one of the following decimals aligned to one of the fractions in the number line estimation task (e.g., 5/6 = 0.83): 0.053, 0.10, 0.14, 0.25, 0.375, 0.41, 0.5, 0.57, 0.625, 0.83, 0.87, 0.92). Each student was assigned a single score for their total correct (out of 12).

Proportional Reasoning

The "Goldilocks" proportional equivalence task developed by Begolli et al. (2020), which was adapted from Boyer and Levine (2012), was used to assess student's understanding of proportion. In this task (see Figure 1), students are told that Goldilocks likes her chocolate milk to taste "just right." They are shown a target column that represents Goldilocks' chocolate milk, comprised of brown "chocolate" and white "milk" sections. Students are asked which of two response options would taste the same as Goldilocks (i.e., which has the same chocolate to milk ratio as the target column). The target ratios (scaling up and down) and foils (matching denominator or numerator) followed the same procedures as Begolli et al. (2020) and Boyer and Levine (2012). Students were randomly assigned to one of three versions of the proportional equivalence task, containing continuous, discretized, or discrete items (Figure 1). The proportional equivalence task was administered on a touchscreen tablet.

Figure 1

Example Item (Ratio: 1/4) From the Proportional Reasoning Task by Condition

Note. Taken from Begolli et al. (2020), which was adapted from Boyer and Levine (2012).

Whole Number Computation

The mathematics subsection of the Wide Range Achievement Test (WRAT; Wilkinson & Robertson, 2006) was used to assess whole number computation skills. The WRAT is normed for people aged 5 through 94, with high internal reliability (.86 to .90 for a grade-based sample) and validity (correlates with other broad measures of mathematics knowledge). The mathematics subsection is comprised of a series of whole number-based computation problems (addition, subtraction, multiplication, and division). While a few items include fractions and decimals, these items are located towards the end of the assessment and were not attempted by any of the students in the current study. Notably, the mathematics subsection of the WRAT is a standardized test used clinically (Pearson) and in scholarly research in both countries to capture general mathematics achievement (e.g., Cheong et al., 2017; Rohde & Thompson, 2007) and has high concurrent validity with other broader measures of mathematics achievement (e.g., Abreu-Mendoza et al., 2019). The WRAT was administered using paper and pencil. Scores were the total correct within the time limit.

Procedure

The researcher administered the assessments in a group setting at the classroom level. Students completed the assessments in the following fixed order: fraction number line estimation, proportional reasoning, decimal comparison, and then whole number computation.

Results

Differences Between Australian and U.S. Students

An a priori power analysis using G*Power (Faul et al., 2007), with α = .05, 1-β = .80, numerator *df* = 1, and groups = 2 for ANOVA, suggests a total sample of 128 is required to detect a medium effect size of f = .25. Larger effect sizes (f > .4) are expected based on previous research. With 30-39 students per grade/country combination, Torbeyns et al. (2015) found effect sizes ranging from f = .5-.62 (η_p^2 = .20 - .28) for fraction number line estimation and f = .2-.25 (η_p^2 = .04 - .06) for fraction comparison. With 19-26 children per age group, Möhring et al. (2018) found effect sizes of f = .98 (η_p^2 = .49) for fraction number line estimation, f = .52 (η_p^2 = .21) for proportional reasoning, and f = .47 (η_p^2 = .18) for spatial scaling. Subsequently, the current sample of 156 students is sufficient to detect the expected medium to large effect sizes.

Fraction Number Line Estimation

Separate two (country) by two (grade) ANOVAs were conducted to examine differences in 0-1 and 0-5 number line estimation. There were significant main effects of country: Australian students were more accurate than U.S. students on the 0-1 number line estimation task, $F(1,151) = 8.68$, $p = .004$, $\eta_p^2 = .05$, and the 0-5 number line estimation task, $F(1,151) = 5.33$, $p = .022$, $\eta_p^2 = .034$. There were also significant main effects of grade: Students in sixth grade were more accurate than students in fourth grade on the 0-1 number line estimation task, $F(1,151) = 10.77$, $p = .001$, $\eta_p^2 = .07$, and the 0-5 number line estimation task, $F(1,151) = 14.06$, $p < .0001$, $\eta_p^2 = .09$. There were no significant interaction effects, $p = .602$, $\eta_p^2 = .007$.

Decimal Comparison

A two (country) by two (grade) ANOVA was conducted to examine differences in decimal comparison. There was no significant effect of country, $F(1,151) = .42$, $p = .91$, $\eta_p^2 = .00$, but there was a main significant effect of grade: students in sixth grade scored higher than students in fourth grade, $F(1, 151) = 39.49$, $p < .0001$, $\eta_p^2 = .24$. There was no significant interaction effect, $p = .13$, $\eta_p^2 = .02$.

Proportional Reasoning

A two (country) by two (grade) by three (version type) ANOVA was conducted to examine differences in proportional reasoning. There was a significant main effect of country: with Australian students scoring higher than U.S. students, $F(1,52) = 5.174$, $p = .024$, $\eta_p^2 = .033$, and grade: with sixth grade students scoring higher than fourth grade students, $F(1,151) = 4.851$, $p = .029$, $\eta_p^2 = .033$. Because there was not a significant main effect of version type, $p = .882$, $\eta_p^2 = .002$ or interaction terms (p's > .05), all subsequent analysis did not consider version type.

Whole Number Computation

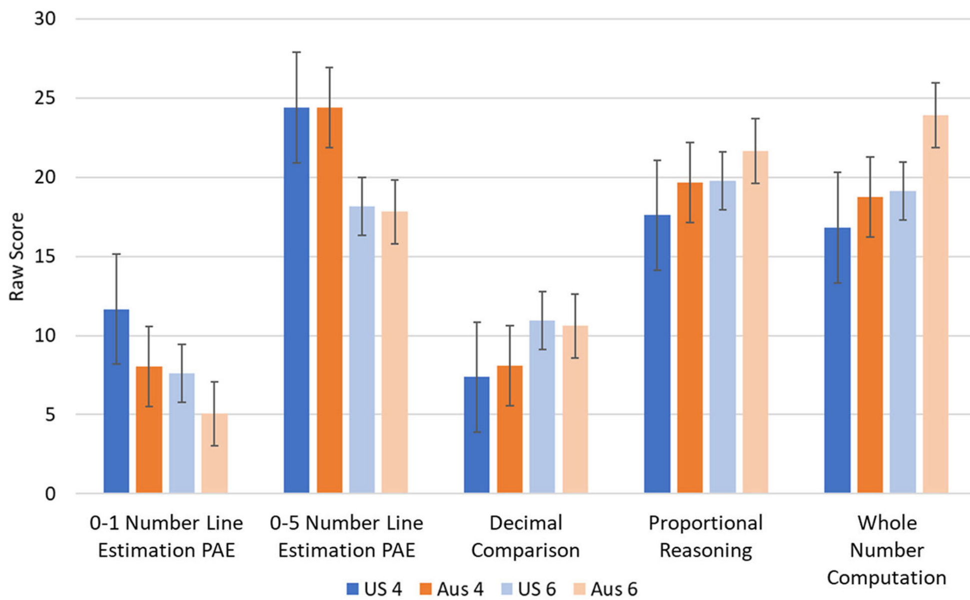
A two (country) by two (grade) ANOVA was conducted to examine differences in performance on the whole number computation. There was a significant main effect of country: Australian students scored higher than U.S. students, $F(1, 151) = 71.88$, $p < .001$, $\eta_p^2 = .36$, and a significant main effect of grade: students in sixth grade scored higher than students in fourth grade, $F(1,151) = 26.61$, $p < .001$, $\eta_p^2 = .15$. There was no significant interaction effect, $p = .108$, $\eta_p^2 = .018$.

Summary

Australian students significantly outperformed U.S. students on fraction number line estimation, proportional reasoning, and whole number computation. There were no significant country level differences on decimal comparison. See Figure 2 for mean performance by country and grade on all outcome measures.

Figure 2

Mean and Standard Deviation (Error Bars) by Country and Grade



Relation Among Fraction Number Line Estimation, Decimal Comparison, Proportional Reasoning, and Whole Number Computation

To examine cross-national differences in the relative contributions of fraction number line estimation, decimal comparison, and proportional reasoning to whole number computation, we conducted hierarchical linear regression analyses. When Torbeyns et al. (2015) completed their cross-national study, they created separate regression models for each grade/country cohort (resulting in four models). An omnibus approach involves creating a single model that includes main effects, interaction terms (by country and by grade), and high-order interaction terms (by country and grade). Unfortunately, the current study does not have enough power to include both interaction and higher-order interaction terms into a single model. As a compromise between these two approaches, we decided to create separate models for each grade (resulting in two models), so that we could examine interactions by country. This choice was driven by strong theory; the bulk of fraction instruction occurs between fourth and sixth grade (see introduction), and so grade would likely account for a lot of variance unrelated to our fundamental research question. Our aim is to examine cross-national differences in each grade. An a priori power analysis using G*Power (Faul et al., 2007), with α = .05, 1-β = .80, and seven predictor variables for multiple regression, suggests a total sample of 102 is required to detect a medium effect size of f^2 = .15.

Fourth Grade Sample

See Table 1 for correlations. For both countries, 0-1 fraction number line estimation, decimal comparison, and whole number computation are correlated. In Australia, 0-5 fraction number line estimation is not correlated with proportional reasoning accuracy or whole number computation. In the US, proportional reasoning is not correlated with whole number computation.

Table 1

Fourth Grade Correlations Among Number Line Estimation Tasks, Proportional Reasoning, and Whole Number Computation by Country

Task	Decimal Comparison Accuracy	Proportional Reasoning Accuracy	Whole Number Computation
Australia			
0-1 Fraction Number line PAE	.44**	.48**	.41**
0-5 Fraction Number line PAE	.41*	.05^ns	.19^ns
Composite Fraction Number Line PAE	.52**	.37*	.41**
Decimal Comparison Accuracy	–	.40*	.61**
Proportional Reasoning Accuracy		–	.31*
Whole Number Computation			–
United States			
0-1 Fraction Number line PAE	.35*	.55**	.39*
0-5 Fraction Number line PAE	.71**	.50**	.57**
Composite Fraction Number Line PAE	.57**	.60**	.54**
Decimal Comparison Accuracy	–	.42*	.39*
Proportional Reasoning Accuracy		–	.25^ns
Whole Number Computation			–

*p < .05. **p < .01. ***p < .001, ns > .05.

Multiple regression analyses with enter method was used to predict whole number computation. Regression assumes homogeneity of slopes, which is assessed through the inclusion of interaction terms (McDonald, 2009). When interaction terms are nonsignificant, it is recommended they be removed from the model and the analysis rerun (Engqvist, 2005; Goldberg & Scheiner, 2001; Lorah, 2020; Quinn & Keough, 2002). Our model initially included the following predictor variables: country, fraction number line estimation, decimal comparison, and proportional reasoning as well as interaction terms by country. This overall model explained 52% of the variance in whole number computation, $F(7, 60) = 11.39$, $p < .001$, AIC = 167.73. There was an interaction between decimal comparison and country, with decimal comparison predicting whole number computation for Australian students and not US students (Table 2). No other interaction terms were significant.

Table 2

Regression Models Predicting Whole Number Computation in Fourth Grade

Predictor	*b*	*SE*	*t*	*p*
Model 1 (Including nonsignificant and significant interaction terms)				
Country	.71	5.05	1.40	.17
Fraction number line estimation (FNLE)	.45	.29	1.54	.13
Decimal comparison	.30	.28	1.08	.29
Proportional reasoning	.38	.27	1.38	.17
FNLE*country	.16	.20	.84	.41
Decimal comparison*country	.38	.19	1.96	.05
Proportional reasoning*country	.31	.18	1.77	.08
Model 2 (Including only significant interaction terms)				
Country	.72	2.28	.32	.75
Fraction number line estimation (FNLE)	.14	.07	1.97	.05
Decimal comparison	.59	.41	1.42	.16
Proportional reasoning	.08	.08	.99	.32
Decimal comparison*country	.71	.28	2.58	.01

Because the interaction terms between country and fraction number line estimation and between country and proportional reasoning were nonsignificant, as recommended, we removed these variables and reran the analysis. This final model included the following predictor variables: country, fraction number line estimation, decimal comparison, and proportional reasoning, as well as an interaction term for decimal comparison by country. The results explained 51% of the variance in whole number computation, $F(5, 62) = 14.98$, $p < .001$, AIC = 167.36. There was a significant main effect of fraction number line estimation and an interaction between decimal comparison and country (Table 2).

Sixth Grade Sample

See Table 3 for correlations. For the Australian sample, 0-5 number line estimation and proportional reasoning are correlated with whole number computation, whereas 0-1 fraction number line estimation and decimal comparison is not. For the US sample, all variables were correlated, except proportional reasoning was not correlated with whole number computation.

Table 3

Sixth Grade Correlations Among Number Line Estimation Tasks, Proportional Reasoning, and Whole Number Computation by Country

Task	Decimal Comparison Accuracy	Proportional Reasoning Accuracy	Whole Number Computation
Australia			
0-1 Fraction Number line PAE	.201^{ns}	.384*	.221^{ns}
0-5 Fraction Number line PAE	.398*	.318^{ns}	.480*
Composite Fraction Number Line PAE	.419*	.417*	.458*
Decimal Comparison Accuracy	–	.094^{ns}	.316^{ns}
Proportional Reasoning Accuracy		–	.457*
Whole Number Computation			–
United States			
0-1 Fraction Number line PAE	.410**	.599**	.506**
0-5 Fraction Number line PAE	.465**	.464**	.501**
Composite Fraction Number Line PAE	.498**	.589**	.571**
Decimal Comparison Accuracy	–	.348*	.348*
Proportional Reasoning Accuracy		–	.288^{ns}
Whole Number Computation			–

*$p < .05$. **$p < .01$. ***$p < .001$, ns > .05.

A multiple regression was conducted for the sixth-grade sample following the same parameters as the fourth-grade sample. With the inclusion of all interaction terms, the overall model explained 62% of the variance, $F(7, 60) = 16.62$, $p < .001$, AIC = 163.45. There was an interaction between proportional reasoning and country, with proportional reasoning predicting whole number computation for Australian students and not US students (see Table 4). No other interaction terms were significant.

Because the interaction terms between country and fraction number line estimation and between country and decimal comparison were nonsignificant, as recommended, we removed these variables and reran the analysis. This final model included the following predictor variables: country, fraction number line estimation, decimal comparison, and proportional reasoning, as well as an interaction term for proportional reasoning by country. The results explained 63% of the variance in whole number computation, $F(5, 62) = 23.68$, $p < .001$, AIC = 160.13. There were significant main effects of fraction number line estimation and proportional reasoning, and an interaction between proportional reasoning and country (Table 4).

Table 4

Multiple Regression Model Predicting Whole Number Computation in Sixth Grade

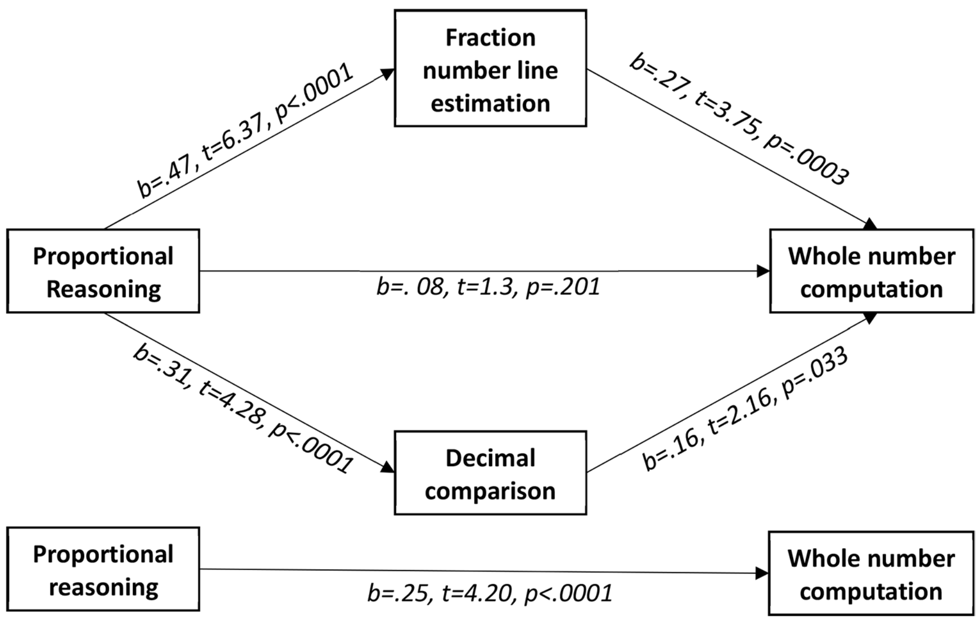
Predictor	*b*	*SE*	*t*	*p*
Model 3 (Including nonsignificant and significant interaction terms)				
Country	.00	6.29	.00	1.00
Fraction number line estimation (FNLE)	.14	.23	.60	.55
Decimal comparison	.13	.66	.20	.85
Proportional reasoning	.43	.24	1.84	.07
FNLE*country	.12	.17	.73	.47
Decimal comparison*country	.008	.40	.02	.99
Proportional reasoning*country	.39	.17	2.29	.03
Model 4 (Including only significant interaction terms)				
Country	2.18	3.39	.64	.52
Fraction number line estimation (FNLE)	.29	.07	3.91	.001
Decimal comparison	.16	.19	.81	.42
Proportional reasoning	.50	.22	2.33	.02
Proportional reasoning*country	.43	.16	2.76	.01

The Relation Between Proportional Reasoning and Whole Number Computation Is Fully Mediated by Fraction Number Line Estimation and Decimal Comparison

To examine if the relation between proportional reasoning and whole number computation is mediated by fraction number line estimation or decimal magnitude comparison, we conducted a parallel mediation analysis using the PROCESS macro (version 3.5) on SPSS (Hayes, 2017) following the data analytic procedure suggested by Hayes (2009, 2017). Because calculating power for mediation analyses is not straightforward (Schoemann et al., 2017), we have used Fritz and MacKinnon's (2007) published guidelines. Using $\alpha = .05$, $1\text{-}\beta = .80$, Fritz and MacKinnon (2007) suggest a sample size 116 to 148 to have sufficient power to detect different combinations of small to medium effects of the predictor variable on the mediator and the mediator on the outcome variable. Subsequently, we examined the sample as a whole to have sufficient power, controlling for country and grade by including them as co-variates in the model. In addition, 95% bias-corrected confidence intervals based on 10,000 bootstrap samples were used. The results indicated that the relation between proportional reasoning and whole number computation is entirely mediated by fraction number line estimation and decimal comparison (Figure 3). This model accounts for 50% of the variance predicted whole number computation.

Figure 3

Mediation Analysis Showing the Relation Between Proportional Reasoning and Whole Number Computation Is Fully Mediated by Fraction Number Line Estimation and Decimal Comparison



Discussion

Although a growing body of work has found that the ability to reason about rational magnitude supports mathematics learning (e.g., Bailey et al., 2012; Resnick et al., 2016; Siegler & Pyke, 2013), this research has primarily taken place within the United States context and has not considered the potential roles of decimal magnitude (Resnick et al., 2019) or proportional reasoning. The current study provides cross-national evidence by investigating the relations between fraction and decimal magnitude understanding, proportional reasoning, and whole number computation (as a proxy for mathematics achievement) within Australia and the United States. By comparing two countries that are similar on many important factors (e.g., language, Western culture, educator's mathematics background and understanding, and fraction learning sequence and progression), but differ on their inclusion of measurement models to teach fractions, we aimed to also examine the influence of fraction instruction on overall performance and on the relation between rational magnitude understanding and mathematics achievement.

Differences Between Australian and U.S. Students

In the current study, Australian students outperformed students from the United States on fraction number line estimation. This finding is aligned with Torbeyns et al. (2015), where students from China and Belgium also outperformed students from the United States on fraction number line estimation. While many cultural and educational factors may have contributed to Torbeyns et al.'s (2015) findings, the countries in the current study were more similar. A key difference between Australian and United States fraction instruction is the inclusion of measurement models versus focusing entirely on a part-whole model, respectively. Notably, our findings may be due to Australian curriculum using multiple representations (as opposed to the influence of measurement models specifically). Nevertheless, the current study shows that measurement model approaches, naturally integrated into wider curriculum at the classroom level, may improve the understanding of rational magnitude.

Relation Among Fraction Number Line Estimation, Decimal Comparison, Proportional Reasoning, and Whole Number Computation

Consistent with existing literature (e.g., Bailey et al., 2012; Resnick et al., 2016; Siegler & Pyke, 2013; Siegler et al., 2012; Torbeyns et al., 2015), a key finding from the current study was that rational magnitude understanding was predictive of whole number computation across Australia and the United States. However, the specific relations varied across country and grade. Although fraction number line estimation was predictive of whole number computation across all cohorts, we found that decimal comparison provided unique variance for Australian fourth grade students whereas proportional reasoning provided unique variance for Australian sixth grade students compared to their US peers. This finding is not aligned with previous work that assumes the same foundational role(s) of fraction and decimal magnitude across all contexts (DeWolf et al., 2015; Resnick et al., 2019; Siegler & Lortie-Forgues, 2014).

One reason decimal magnitude understanding may be more important in the Australian fourth grade context, is that decimals play a large role in understanding the metric system, which is Australia's official measurement standard and used across all industries. While the United States national standards includes learning metric units, they also include learning customary units (e.g., feet and inches; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and the United States government has not officially adopted the metric system (e.g., regulatory speed limits are posted in miles per hour). Notably, both countries may engage with decimals when learning about currency.

In contrast, Australian sixth grade students, after receiving the bulk of fraction instruction that includes emphasis on a measurement model, may be better equipped to utilize proportional reasoning and spatial scaling strategies when completing mathematics problems. Measurement model instruction may change the spatial representations (more accurate linear representation) and/or strategy (spatial scaling strategies) underlying both proportional reasoning and rational number problem-solving. The stronger relations among these measures are consistent with a common underlying change in the Australian students spatial and numerical thinking (though further research is needed to discriminate between this explanation and alternatives). In contrast, in early United States fraction instruction, fourth grade students tend to use whole number properties as a strategy to reason about fraction magnitude (Rinne et al., 2017). Eventually, some children adopt transitional/partial understandings on their way to developing a holistic representation of magnitude by sixth grade (Rinne et al., 2017). Although Australian fourth grade classrooms focus on measurement models, students may still use rule-based strategies and not spatial scaling strategies.

Finally, the findings from the current study suggest that a key mechanism for how reasoning about magnitude (including rational numbers) supports mathematic achievement is proportional reasoning. The relation between proportional reasoning and whole number computation is fully mediated by rational magnitude understanding, suggesting that proportional reasoning may support the development of an accurate spatial representation of magnitude that can be flexibly and proportionally scaled, which in turn supports children's mathematics learning. Indeed, using a holistic analog representation to reason about magnitude requires imaging the internal proportion shrinking or growing. Experimental studies, however, are required to determine causal effects.

Limitations and Future Directions

Although the current study contrasted two countries that are similar on many important factors, it is impossible to align all national, cultural, and education contexts. Subsequently, there may be other factors that influence fraction and decimal magnitude learning and their relation to mathematics achievement. For example, Australian students outperformed the United States students on mathematics achievement, which contrasts with the TIMSS (Fishbein et al., 2021) and PISA (OECD, 2019) reports. While this may simply be due to the TIMSS (curriculum-based), PISA (application-based), and WRAT (computation) assessing different mathematics skills, it could also reflect interactions between unobserved cultural and educational factors (e.g., the sample being from a higher SES population combined with using measurement models of teaching fractions), limiting the generalizability of the findings. However, reasoning about fraction and decimal magnitude is predictive of mathematics achievement, and proportional reasoning supports reasoning about magnitude, in a wide range of SES populations, including low SES (e.g., Hansen et al., 2015; Resnick et al., 2016, 2019). Subsequently, we hypothesize that the observed country-level differences would be even larger within low SES populations.

Differences between the Australian and US students on the WRAT could also be explained by this sample of Australian students happening to have superior math skills. We do not believe this to be the case because we matched the samples based on a range of demographic, cultural, and educational factors. Nevertheless, even if the samples had different overall mathematics skills, that does not change our finding that reasoning about rational magnitude and proportional reasoning supports whole number computation cross-nationally. Further, country is not predictive of whole number computation in the final regression model, emphasizing the relative importance of rational magnitude and proportional reasoning over any country-level differences that may exist. It seems likely that higher WRAT scores may be a consequence of having higher fraction and decimal magnitude understanding and proportional reasoning skills (Ye et al., 2016).

It may also be the case that fraction instruction varies within each country, including aspects not identified within this study (e.g., how much time is spent on rational number instruction). Future work should consider a wider range of cultural and educational settings in exploring how and when reasoning about different kinds of magnitude support mathematics achievement. In addition, future research should also consider the role of other individual differences (e.g., SES, working memory, gender, other spatial skills).

Another consideration concerns the proportional reasoning measure, which was presented in three versions: continuous, discretized, and discrete. We originally chose to assign participants to a single version, because completing one type of proportional reasoning task can influence strategies on subsequent tasks (Boyer & Levine, 2015). However, our final sample was not large enough to consider the role of different proportional reasoning strategies. Because there were no significant effects of version type, nor were there any interactions, and because the same proportion of students from Australia and the United States took the three measures (meaning that any noise from having the three versions was equally distributed across the country and grade cohorts), we did not include version type in subsequent analyses. Subsequent work with larger samples or more sessions could explore the role of the various kinds of proportional reasoning strategies in more depth. We recommend future work focus on the role of spatial scaling strategies specifically given the spatial nature of magnitude representation.

Notably, a larger sample size would have provided our study with more power to observe potentially present smaller effects. Indeed, the inclusion of interaction terms in our models further reduced our power. However, we observed large effect sizes in both countries indicating that rational magnitude understanding is important cross-nationally, but the specific relation varying between country, which was accounted for by proportional reasoning when put together in a single model. While our sample size may not have been sufficient to detect if a small effect were present, having no effect versus a small effect in this instance would not change the interpretation – there is a differential relation between math and rational number in Australia compared to the USA.

Conclusions

A key finding from the current study is that rational magnitude understanding was predictive of mathematics achievement across Australia and the United States, even though mathematics achievement was measured using whole number computation problems. This supports the integrated theory of numerical development (Siegler & Lortie-Forgues, 2014; Siegler et al., 2011), which suggests that a unifying feature of all real numbers is they have a magnitude that can be ordered along the number line, and emphasizes learning about rational numbers (e.g., fractions and decimals) as a key step because they require a reorganization of numerical properties. However, the specific relations varied across the countries and grades, highlighting the influence of teaching practices and the role of proportional reasoning in how magnitude representation may support mathematics achievement. Taken together, the findings from the current study emphasize the potential benefits of including measurement models into everyday fraction instruction, along with the promotion of spatial scaling proportional reasoning strategies to reason about magnitude.

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